University of California at San Diego – Department of Physics – Prof. John McGreevy

Physics 215A QFT Fall 2016
Assignment 3

Due 11am Thursday, October 13, 2016

1. **Momentum.** Starting from the expression for the stress energy tensor we derived for free scalar field theory, find the generator of spatial translations $\vec{\mathbf{P}}$. Show that a state of n particles of definite wavenumber

$$\mathbf{a}^\dagger_{\vec{k}_1} \cdots \mathbf{a}^\dagger_{\vec{k}_n} |0\rangle$$

is an eigenstate of $\vec{\mathbf{P}}$ with a reasonable answer for the eigenvalue.

2. **Vacuum energy from the propagator.**

Consider a scalar field with

$$\mathbf{H} = \int d^d x \, \frac{1}{2} \left(\pi^2 + (\vec{\nabla}\phi)^2 + m^2 \phi^2 \right).$$

Reproduce the formal expression for the vacuum energy

$$\langle 0|\mathbf{H}|0\rangle = V \int đ^d k \frac{1}{2} \hbar \omega_{\vec{k}}$$

using the two point function

$$\langle 0| \phi(x)^2 |0\rangle = \langle 0|\phi(0)\phi(0)|0\rangle = \lim_{\vec{x},t\to 0} \langle 0| \phi(x)\phi(0) |0\rangle$$

and its derivatives. (V is the volume of space.)

Later we will learn to draw this process as a Feynman diagram which is a circle (a line connecting a point to itself).

3. **The identity does nothing twice.**

Check our relativistic state normalization by squaring our expression for the identity in the 1-particle sector:

$$\mathbb{1}_1^2 \overset{!}{=} \mathbb{1}_1 = \int \frac{đ^d p}{2\omega_{\vec{p}}} |\vec{p}\rangle \langle \vec{p}| \,.$$

4. **The retarded propagator is a Green's function.**

Consider the retarded propagator for a real, free, massive scalar field:

$$D_R(x-y) \equiv \theta(x^0 - y^0) \langle 0| [\phi(x), \phi(y)] |0\rangle \,.$$

Show that it is a Green's function for the Klein-Gordon operator, in the sense that

$$\left(\partial_x^2 + m^2\right) D_R(x-y) = a\delta^{d+1}(x-y).$$

Find a.

Use this to generalize the mode expansion of the scalar field to the situation with an external *source*, *i.e.*when we add to the lagrangian density $\phi(x)j(x)$ for some fixed c-number function $j(x)$.