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Quantum Mechanics C (Physics 212A) Fall 2015
Worksheet 8

Announcements

- The 212A web site is:

  http://physics.ucsd.edu/∼mcgreevy/f15/ .

  Please check it regularly! It contains relevant course information!

Problems

1. **Charged Particle on a Ring**

   Consider the quantum mechanics of a particle of mass m and charge q on a circular hoop of radius R in the presence of an external magnetic field perpendicular to the plane of the hoop.

   (You can set all these parameters to 1 if you like. I will adopt units where $\hbar = 1 = c$)

   Let $\phi(t)$ be the angular coordinate of the particle such that $\phi \equiv \phi + 2\pi$

   The action is
   $$S[\phi] = \int \mathrm{d}t \ \frac{m}{2} R^2 \dot{\phi}^2 + (\frac{\theta}{2\pi})\dot{\phi} \tag{1}$$
   The θ term is a "topological term" as we'll see.

   (a) Show that if there is a azimuthally symmetric magnetic field passing through the ring, then θ is related to B by
   $$\theta = q\Phi_B$$
   where Φ_B is the magnetic flux through the ring.

   (b) Show that θ is a periodic parameter, i.e. theories with θ and $\theta \to \theta + 2\pi$ are equivalent.

   (c) Compute the canonical momentum associated with ϕ, and determine the Hamiltonian for this system.

   (d) Show that the θ term doesn't effect the classical equation of motion for ϕ.

   (e) Find the energies and wavefunctions by solving the Schrödinger equation. What happens to the eigenstates $|n, \theta\rangle$ and energies $E_n(\theta)$ as $\theta \to \theta + 2\pi$?

(f) Suppose I do a gauge transformation $A \to A + \nabla\Lambda$ that "removes" the presence of the magnetic field. What Λ accomplishes this? Why does the spectrum not change?

(g) Show that for $\frac{\theta}{2\pi} = \frac{2m+1}{2}$ for $m \in \mathbb{Z}$ the spectrum is two-fold degenerate

Now suppose I turn on an electric field in the xy-plane. $\vec{E} = (E_x \cos\phi, E_y \sin\phi, 0)$

(h) Write the Hamiltonian for the system in the presence of this field

(i) Consider $\frac{\theta}{2\pi} = \frac{1}{2} + \delta$ for $\delta \ll 1$ so that $|0\rangle$ and $|1\rangle$ are nearly degenerate. Write an effective Hamiltonian for this 2-state system, including the electric field.

(j) Calculate the 'precession' frequency of the particle in this field keeping with the 2-state system idea.

(k) Now suppose $E_y = E_x = E\cos\omega t$ varies in time. Suppose at $t = 0$ the particle is in state $|0\rangle$. What's the probability of the particle to be in $|1\rangle$ at a later time T? Show this probability is periodic and find the period T_0. What's a physical constraint on ω for this picture to be valid?