University of California at San Diego – Department of Physics – TA: Shauna Kravec

Quantum Mechanics C (Physics 212A) Fall 2015
Worksheet 7

Announcements

- The 212A web site is:

  http://physics.ucsd.edu/∼mcgreevy/f15/ .

  Please check it regularly! It contains relevant course information!

Problems

1. **Tight Binding for SSH**

   Let's apply our understanding of Bloch's theorem to a simple chain involving 2 types of sites A and B. An example could be the following cartoon:

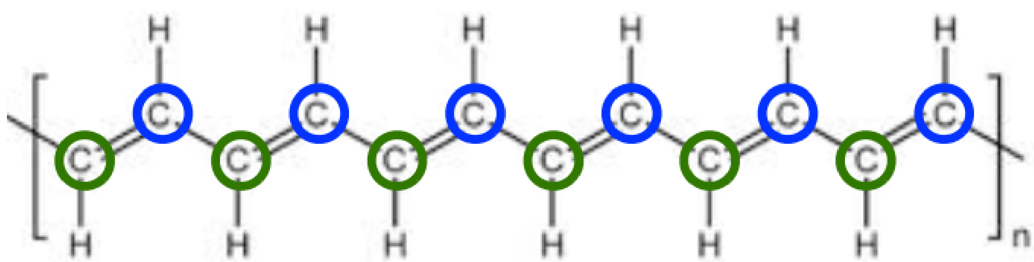

Figure 1: A cartoon of trans-polyacetalyene

Let $j \in \{0, \cdots, N-1\}$ label the number of A/B pairs. The Hilbert space for a particle living on this space is $\mathcal{H} = \text{span}\{|jA\rangle, |jB\rangle\}$ representing occupation of that site.[1]

Now suppose that the particle is allowed to hop with a pair with energy cost t_1 and out of the pair with energy cost t_2. So the difference between crossing a single versus double bond above.

(a) Write a Hamiltonian for the system. Use the notation $v_j \equiv \begin{pmatrix} \langle jA| \\ \langle jB| \end{pmatrix}$ to simplify

Notice the $j \to j+1$ symmetry. This translation symmetry suggests we can define a crystal momentum k from our discussion of Bloch's theorem.

[1] Assume periodic boundary conditions $|NA\rangle \equiv |0A\rangle$

(b) Define the Fourier transform $v_j = \frac{1}{\sqrt{N}} \sum_k e^{\mathbf{i}kj} v_k$ and use this to write the Hamiltonian as $H = -\sum_k v_k^\dagger H(k) v_k$. What is $H(k)$?

(c) Diagonalize $H(k)$ to find the energies E_k. You should get two eigenvalues or *bands*. What happens when $t_1 = t_2$?

So what we have is a model of an insulator with two distinct *phases*. One where $t_1 > t_2$ and another where $t_2 > t_1$. The bands cross at the *phase transition*. Is there another way to connect the regions to avoid the phase transition?

(d) Consider the following projectors $P_A \equiv \sum_i |iA\rangle\langle iA|$ and $P_B \equiv \sum_i |iB\rangle\langle iB|$ to make the operator: $\Sigma \equiv P_A - P_B$. Show that Σ is unitary and hermitian

(e) Show that $\Sigma H(k) \Sigma = -H(k)$ it is a *chiral* symmetry. What does this imply for the spectrum of $H(k)$?

(f) Apply your knowledge of avoided crossing to answer: what sort of term should you add to $H(k)$ to connect the eigenvalues?

How does this term transform under Σ?

In this way the model is *protected* by the symmetry Σ. You can only connect the two phases by breaking the symmetry or going through a phase transition.

So what makes these phases different?

(g) Define $q(k) \equiv \frac{h(k)}{|E_k|}$ for $H(k) = \begin{pmatrix} 0 & h(k) \\ h^\dagger(k) & 0 \end{pmatrix}$. Convince yourself this is a map from the circle to the circle.

(h) Define the *winding number* of $q(k)$ to be $\nu \equiv \frac{\mathbf{i}}{2\pi} \int \mathrm{d}k \; \frac{1}{q} \partial_k q$. What is it for the different values of (t_1, t_2) ?

This is a *topological* difference between the phases; the winding number can't change continuously.