University of California at San Diego – Department of Physics – TA: Shauna Kravec

Quantum Mechanics C (Physics 212A) Fall 2015
Worksheet 2

Announcements

- The 212A web site is:

  http://physics.ucsd.edu/∼mcgreevy/f15/ .

  Please check it regularly! It contains relevant course information!

Problems

1. **Formaldehyde** (From Le Bellac)

   Let's consider a simple two state system motivated by the Huckel theory.

   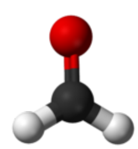

   Figure 1: Formaldehyde visualized

   There are two π-electrons associated with the double bond between carbon and oxygen. Let's consider the Hilbert space of a single π-electron as $\mathcal{H} = \text{span}\{|O\rangle, |C\rangle\}$ where these represent occupation on either the carbon or oxygen.

   (a) Give a physical motivation for the Hamiltonian to be of the form

   $$\hat{H} = E_O|O\rangle\langle O| + E_C|C\rangle\langle C| - A(|C\rangle\langle O| + |O\rangle\langle C|) \tag{1}$$

   where $E_C > E_O$ are the energies associated with being localized and A is known as the "delocalization" energy

   (b) Calculate the eigenvalues and eigenvectors associated with (1). Sketch how this would look in position space.

   Assume that the system is in its ground state.

   (c) For a given π-electron, calculate the probability of finding it localized at the oxygen.

   (d) Assume that the electric dipole moment of formaldehyde only gets contributions from the symmetric axis. Express this as a function of the bond length ℓ.

2. **Single Qubit Gates** (From Nielsen-Chuang)

Aside from the usual Pauli matrices there are a few common operators for a two state system. These are the Hadamard (H), the phase gate (S), and the T-gate (T). In the Z-basis these can be written as:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \quad S = \begin{pmatrix}1 & 0\\ 0 & \mathbf{i}\end{pmatrix} \quad T = \begin{pmatrix}1 & 0\\ 0 & e^{\mathbf{i}\frac{\pi}{4}}\end{pmatrix} \tag{2}$$

(a) Write these in terms of our original Pauli's. Note that $S = T^2$. What is the action of H on Z-eigenvectors?

(b) Prove the following identities

$$HXH = Z \quad HYH = -Y \quad HZH = X \tag{3}$$

(c) Show that[1] $T = U_z(\frac{\pi}{4})$ and $HTH = U_x(\frac{\pi}{4})$ where $U_n(\theta) \equiv e^{-\mathbf{i}\frac{\theta}{2}\hat{n}\cdot\vec{\sigma}}$

3. **Quis Custodiet Ipsos Custodes?** (From Jacobs)

Projective measurements lead to some weird things.

Consider a two state system with basis vectors $\{|0\rangle, |1\rangle\}$. We are going to evolve the system according the Hamiltonian $\hat{H} = \frac{\omega}{2}Y$ where Y is the Pauli matrix $\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$.

(a) What is the unitary operator associated with time evolution? Given an initial prepared state of $|\psi_0\rangle = |0\rangle$. Write an expression for $|\psi(t)\rangle$.

(b) What is the probability, as function of time, to measure $|0\rangle$?

(c) Suppose we study the system over the time interval $[0, T]$ where $T \gg \delta t \equiv \frac{T}{N}$. We perform a measurement, in this basis, at every time $\frac{T}{N}, \frac{2T}{N}, \cdots$ where N is large. Assuming each measurement is independent from the other, what's the probability that the spin *never* flips to $|1\rangle$?

(d) Evaluate this probability in the limit of $N \to \infty$.

This is called the *quantum Zeno effect*.

[1] Up to a global phase