Hyperentanglement for advanced quantum communication

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ABSTRACT

Quantum entanglement is known to enable otherwise impossible feats in various communication protocols, such as quantum key distribution and super-dense coding. Here we describe efforts to further enhance the usual benefits, by incorporating quantum states that are simultaneously entangled in multiple degrees of freedom – "hyperentangled". Via the process of spontaneous parametric down conversion, we have demonstrated photon pairs simultaneously entangled in polarization and spatial mode, and have used these to realize remote entangled state preparation, full polarization Bell-state analysis, and the highest reported capacity quantum dense coding.

Keywords: Quantum Communication, Entanglement, Hyperentanglement

1. HYPERENTANGLEMENT

As is now well known, entanglement is a resource for quantum information processing, and entanglement of photons is central to most quantum communication protocols, e.g., dense coding, teleportation, and secure quantum cryptography. Much research has already been done on optical entanglement in a *single* degree of freedom –polarization,[1–4] energy,[5–8] time-bin,[9] momentum,[10] and orbital angular momentum.[11] The phenomenon of "hyperentanglement"[12] – simultaneous quantum correlations in multiple degrees of freedom– is now also being explored as a resource for quantum information processing.[13,14] Here we report on our demonstration of hyperentangled photons from spontaneous parametric down conversion,[15] and their application to two quantum communication protocols: remote entangled state preparation[16] and quantum super dense coding.[17,18]

Hyperentangled photons –simultaneously entangled in multiple degrees of freedom– present a number of unique opportunities in quantum information processing. First, they reside in an enlarged Hilbert space compared to, e.g., that of photons simply entangled in polarization*. For example, for photons entangled in polarization and an effective two-state spatial mode, the Hilbert space is $2 \times 2 \times 2 \times 2 = 16$ dimensional. One advantage of using such a system is that it is relatively easy to perform quantum logic *between* qubits residing in different degrees of freedom of the same photon,[14,21] as opposed to qubits residing in different photons. Consequently, such hyperentanglement enables new capabilities in quantum information processing, including, as we discuss here, remote preparation of entangled states, full Bell-state analysis, and improved super-dense coding, as well as the possibility of quantum communication with larger alphabets.[22,23]

Our source of entangled photons is based on the process of spontaneous parametric down conversion in two adjacent non-linear crystals (BBO). The crystals are oriented with their optic axes in perpendicular planes; type-I phase matching leads to the production of horizontally polarized pairs of photons from the first crystal (arising from the vertical polarization component of the pump laser) and vertically polarized pairs of photons from the second crystal (arising from the horizontal polarization component of the pump). If we pump the crystals so that the amplitudes of these two processes are equal, and arrange the thickness of the crystals to be much less than

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*Of course, the number of degrees of freedom associated with the photons is always fixed, so in one regard the size of the relevant Hilbert space is also pre-determined. However, in most photon experiments, the *interesting* quantum effects are limited to a single degree of freedom, and one ignores the others (or even works to eliminate correlations to the other degrees of freedom[19,20]).

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the diameter of the pump beam, the two processes are indistinguishable and a maximally polarization-entangled state results † $|HH\rangle + e^{i\phi}|VV\rangle$.

We have previously shown that the photons from down conversion can be prepared in a state displaying simultaneous entanglement in polarization, spatial mode, and energy-time,[15] but our experiments here involve only the first two of these degrees of freedom. Figure 1(a) shows a setup for producing and characterizing the hyperentanglement. The photon pairs produced are approximately in the state

$$\underbrace{(|HH\rangle + |VV\rangle)}_{\text{polarization}} \otimes \underbrace{(|lr\rangle + \alpha|gg\rangle + |rl\rangle)}_{\text{spatial modes}}, \tag{1}$$

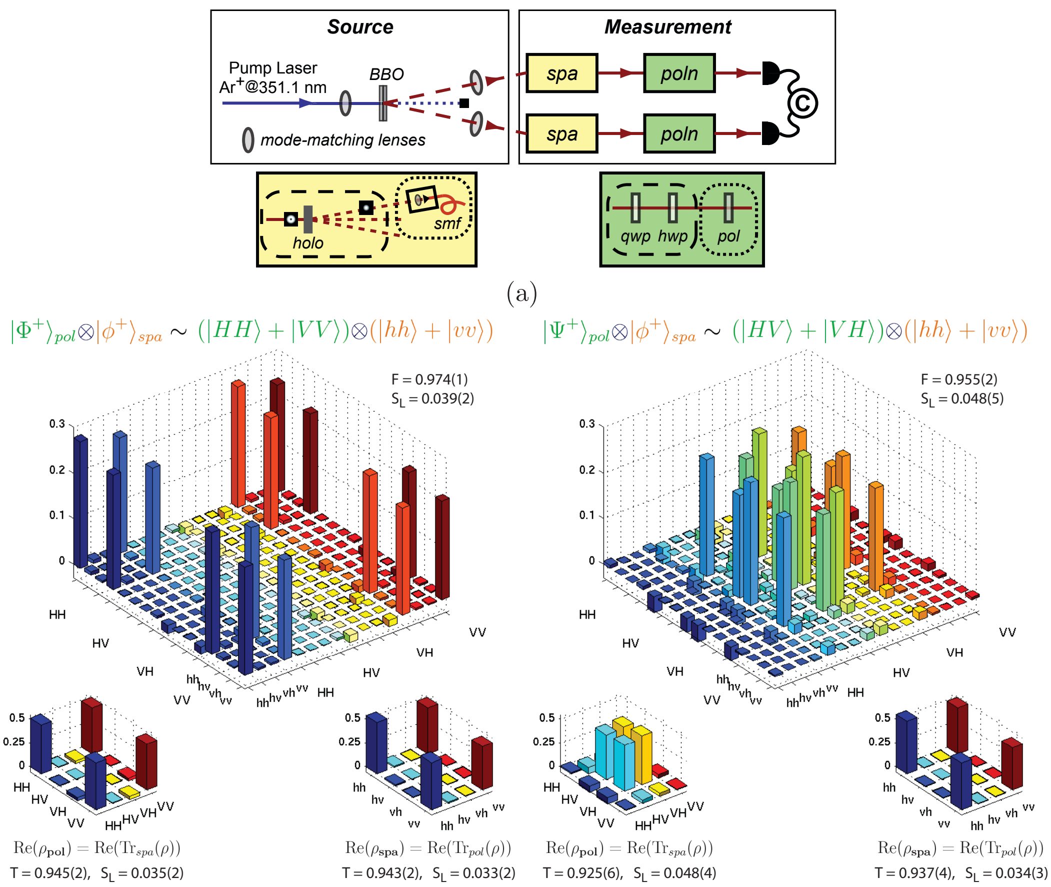
where H(V) represents the horizontal (vertical) photon polarization; $|l\rangle$, $|g\rangle$, $|r\rangle$ represent the Laguerre-Gauss modes carrying $-\hbar$, 0 and $+\hbar$ orbital angular momentum (OAM),[24] respectively, and α describes the OAM spatial-mode balance prescribed by the source and selected via the mode-matching conditions. For many of our experiments, we post-select spatial modes with $\alpha = 0$. The analysis and concomitant post-selection of spatial mode is performed using holographic filters and single-mode fibers. Specifically, the holograms are constructed so that a particular input spatial mode is converted into a Gaussian spatial mode in the first-order diffraction direction. The single-mode optical fiber can then accept this spatial mode. All other spatial modes incident on the diffraction grating are converted in the first-order diffraction direction into non-Gaussian spatial modes, which cannot be collected by the optical fiber. By using various patterned holograms, one can thus project into different spatial-mode states. In this restricted case, our spatial mode states are in OAM Bell-type states. For example, we can readily produce states of the form $|\Phi^{\pm}\rangle \otimes |\phi^{+}\rangle$ and $|\Psi^{\pm}\rangle \otimes |\phi^{+}\rangle$, where $|\Phi^{\pm}\rangle = (|HH\rangle \pm |VV\rangle)/\sqrt{2}$, $|\Psi^{\pm}\rangle = (|HV\rangle \pm |VH\rangle)/\sqrt{2}$, and $|\phi^{+}\rangle = (|hh\rangle + |vv\rangle)/\sqrt{2}$ with $|h\rangle = (|l\rangle + |r\rangle)/\sqrt{2}$ and $|v\rangle = (|l\rangle - |r\rangle)/\sqrt{2}$. $|h\rangle$ and $|v\rangle$ are also described as the $HG_{0,1}$ and $HG_{1,0}$ Hermite Gauss spatial modes.

Figure 1(b) shows two such examples of maximally hyperentangled states, which have greater than 95% fidelities with their target states.[15] Also shown are the (real parts of) the density matrices corresponding to the individual entangled degrees of freedom. These results are obtained by tracing out the overall density matrix over the other degree of freedom. In all cases, we observe very high tangle (T) and very low linear entropy (S_L).

2. REMOTE ENTANGLED STATE PREPARATION

As our first application of hyperentanglement-enhanced quantum communication, we consider the protocol of remote state preparation (RSP),[16] by which Alice can remotely prepare an arbitrary state at Bob's location, albeit only probabilistically. The basic resource requirements for RSP are similar to those for quantum teleportation,[27] in that Alice and Bob must share prior entanglement. By making a particular measurement on her half of the entangled state, Alice is able to affect the state that Bob receives, given that she obtained a particular outcome. RSP is potentially simpler than teleportation, however, because Alice knows what states she wants to send. More importantly, the measurement that Alice needs to make is much simpler, as no Bell-state analysis is required: she simply measures her single photon in a particular basis, and sends a single bit of classical communication (compared to the two bits needed for teleportation) to Bob, indicating whether or not she achieved a positive result. If she did, then Bob's photon should already be in the desired state. If instead Alice recorded the opposite result, then Bob's photon will be prepared into a state orthogonal to the one which Alice desired to send. Unfortunately, due to the fact that it is impossible to perform a universal not gate,[28] there is in general no way that Bob can use this information to "flip" his qubit into the correct state.

†Elsewhere we discuss the fact that the relative phase ϕ in general depends on the precise emission directions of the photons from the birefringent non-linear crystals, which can reduce the entanglement quality if photons are collected over many spatial modes; moreover, we have identified a means by which to compensate for this spatially dependent phase by introducing an extra pair of birefringent elements in the paths of the down conversion photons.[19,20] The net effect is to produce a spatially dependent phase of the opposite sign to that of the down conversion crystals, so that the net phase map is essentially flat, and emitted photons are all produced in the same quantum mechanical state (more precisely, the polarization part of this state is the same). However, in the experiments described here, this spatial compensation technique was not necessary – because we use single-mode fibers to collect the photons, the deleterious effects of this spatially varying phase are negligible.



(b)

Figure 1. (a) An experimental setup to produce and characterize photon pairs simultaneously entangled in polarization and spatial mode. Here we produce degenerate-frequency photons at 702 nm by pumping the adjacent BBO crystals with the 351-nm line from an argon-ion laser. Polarization measurements may be made in an arbitrary basis using a fixed polarizer preceded by adjustable quarter- and half-waveplates. The spatial mode state is analyzed using holographic filters and single-mode fiber collection optics. b. The density matrix (real part) of the detected photons, obtained by a maximum-likelihood analysis.[25,26] The total states have a high fidelity with their intended targets and low linear entropy, while the states associated with the polarization and spatial mode degrees of freedom individually display high tangles. Adapted from Ref. 15

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Using a source of polarization-entangled photons, we have previously implemented the remote state preparation (RSP) protocol, demonstrating that Alice can remotely prepare arbitrary pure and mixed states of Bob's photon with extremely high fidelity.[29] In our present experiment, we go one step further –by using *hyperentangled* photons Alice is able to prepare Bob's photon into an entangled state. In particular, Bob's photon is left entangled between the polarization and spatial mode[‡]. Our experimental setup is shown in Fig. 2(a). As described above, the photons are initially prepared in the hyperentangled state $(|HH\rangle + |VV\rangle) \otimes (|lr\rangle + |rl\rangle)/2$. By projecting onto the state $\cos\theta\langle Hr| + e^{i\phi}\sin\theta\langle Vl|$, Alice is able to prepare Bob's photon into the arbitrary state $\cos\theta|Hl\rangle + e^{i\phi}\sin\theta|Vr\rangle$ (again, this preparation is strictly conditional on Alice's getting a positive result for her measurement, thereby preventing any possibility of superluminal communication with this scheme). In order to project her photon into the entangled basis required, Alice must perform a controlled-not (CNOT) gate between the polarization and spatial-mode degrees of freedom of her photon. She can achieve this using the novel interferometer shown in Fig. 2(a). The first beam splitter of the interferometer is a holographic filter which converts an initial +1 OAM spatial mode into a Gaussian beam in the -1 diffraction order, and converts -1 OAM into a Gaussian in the +1 diffraction order. These two Gaussians (which are subsequently post-selected via single-mode fibers) are then combined on a polarizing beam splitter. Finally, the output ports are analyzed in the diagonal polarization basis. By considering the experimental setup and input state above, one can easily verify that detections at the four possible outputs project the incident photon into one of the four single-photon two-qubit Bell states:

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|Hl\rangle \pm |Vr\rangle\right), \qquad \text{and} \qquad |\psi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|Hr\rangle \pm |Vl\rangle\right).$$

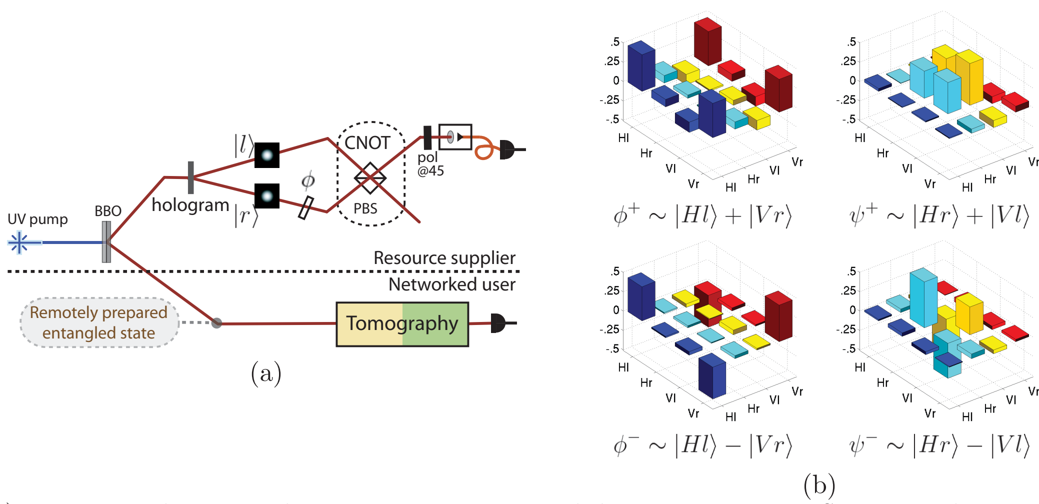


Figure 2. (a) Experimental setup to demonstrate remote entangled-state preparation. Starting with an initially hyperentangled pair of photons, if Alice measures her photon to be in an appropriate state, Bob's photon can be remotely prepared into any desire state of polarization and spatial mode. Specifically, Alice can prepare Bob's photon in a maximally entangled state between these two degrees of freedom, by performing a CNOT gate on her photon. (b) Results of quantum state tomography on Bob's photons for several different target states. In all cases, the fidelity with the desired target state exceeded 90%, and the tangle of the resulting single-photon entanglement averaged 85%.

Conditional on Alice obtaining a particular result, Bob's photon is prepared into an arbitrary entangled state of the polarization and spatial modes. Figure 2(b) shows the tomographically measured density matrices of Bob's remotely prepared entangled states. We have achieved fidelities above 94%, with tangles from 85% to 90%; the main limitations to these results were interferometric phase stability and coupling imbalances. Efforts are now underway to improve these issues, resulting in a more precise remote state preparation.

[‡]While there has been some discussion whether such correlations between two degrees of freedom of a single particle should be described by the term "entanglement", here we invoke the arguments advanced by van Enk,[30] proving that, indeed, entanglement may be present in a single particle, but between two modes.

3. SUPER-DENSE CODING

As a final application of hyperentanglement we now consider the quantum communication protocol of super-dense coding, which in fact was one of the very first quantum information protocols to be proposed.[17] In this protocol Bob desires to send two classical bits of information to Alice, but is only allowed to send her a single photon. As shown in Fig. 3, the protocol requires Alice and Bob to share an entangled pair of photons; by performing only local transformations on his photon, Bob is able to convert the initial entangled state into any one of the four Bell states ($|\phi^{\pm}\rangle$, $|\psi^{\pm}\rangle$). Bob then sends this photon to Alice, who, after performing a full Bell-state analysis (BSA) on the two photons, can infer which of four messages Bob intended to send. The channel capacity of this protocol is therefore $\log_2 4 = 2$ bits per photon. The main difficulty of implementing this protocol in practice is that achieving full BSA using photons is non-trivial. In fact, it has been shown[31,32] that using only linear optics it is not possible to distinguish all four Bell states: only two of the four may be reliably distinguished, with the other two giving an identical experimental signature[§]. The fact that the four Bell states are produced in three distinguishable groups implies that the maximum channel capacity using linear optics is $\log_2 3 \approx 1.58$ bits.

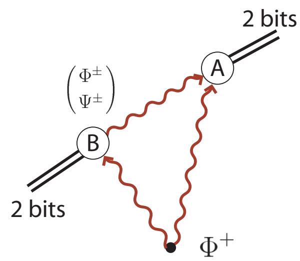


Figure 3. Basic scheme for quantum super dense coding. Alice and Bob share an entangled state. By performing one of four unitary transformations on his particle, Bob can produce any one of the four Bell states. If Alice can then subsequently determine which of the four Bell states the two photons are in, she can determine the two classical bits of information that Bob desired to send her.

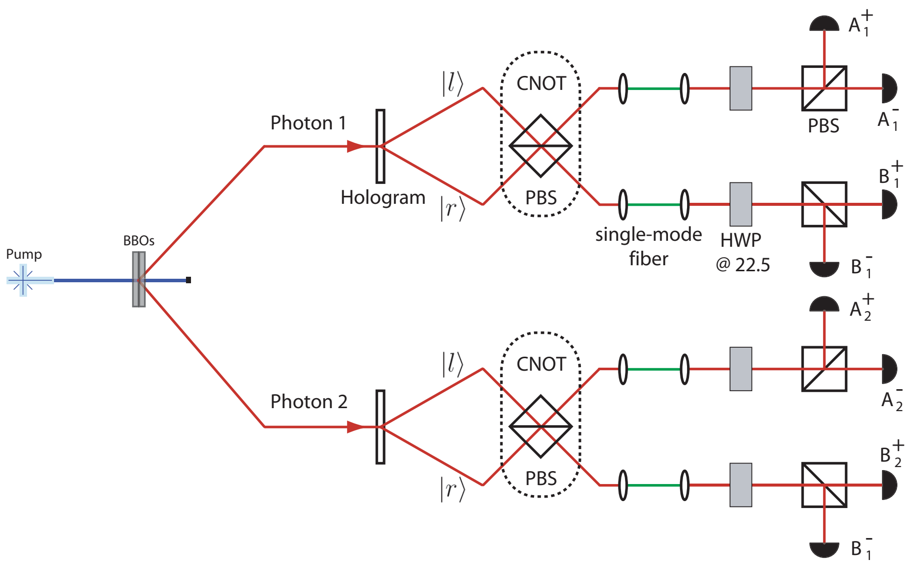
The no-go theorems for linear optics BSA implicitly assumed that the particles are only entangled in a single degree of freedom. However, as was shown nearly a decade ago,[36] a full 100%-efficient Bell-state analysis is possible if the photons are also entangled in another degree of freedom. Such hyperentanglement-assisted BSA has now been experimentally realized,[37,38] but the experimental results were not of sufficient quality to exceed a channel capacity of even 1.3 [¶].

By performing full BSA with photons simultaneously entangled in polarization and spatial mode, we have been able to achieve a dense-coding channel capacity of 1.63, exceeding the "limit" of 1.58 relevant for photons entangled in a single degree of freedom.[18] Our experimental setup is shown in Fig. 4(a). We have implemented a variation of a scheme proposed by Walborn *et al.*,[41] in which the Bell-state analysis may be performed by independent measurements on each of the photons. This is a substantial improvement over the original scheme for hyperentanglement-enhanced BSA,[36,37] which relied on two-photon interference effects. In addition to being more challenging experimentally, the original scheme, when applied to dense coding, required that Alice must store her photon until she receives its transformed partner from Bob; the associated memory requirements would severely limit the present practical application of this protocol. In contrast, in the Walborn et al. scheme Alice can immediately make a measurement on her photon, and simply wait for Bob to send her his photon before making a similar measurement on it. Comparison of the two results then uniquely identifies the polarization Bell state.

Quite conveniently, the experimental apparatus needed for analyzing each of the photons is identical to that presented previously for remote entangled state preparation. In particular, Alice needs to perform a CNOT

[§]There are schemes which can distinguish all four of the Bell States, but these are non-deterministic, and at most 50% efficient.[33,34] Note that if one had sufficiently strong photon-photon interactions, in principle 100%-efficient full BSA would be possible; in practice, however, the necessary interactions are too small by approximately 9 orders of magnitude.[35]

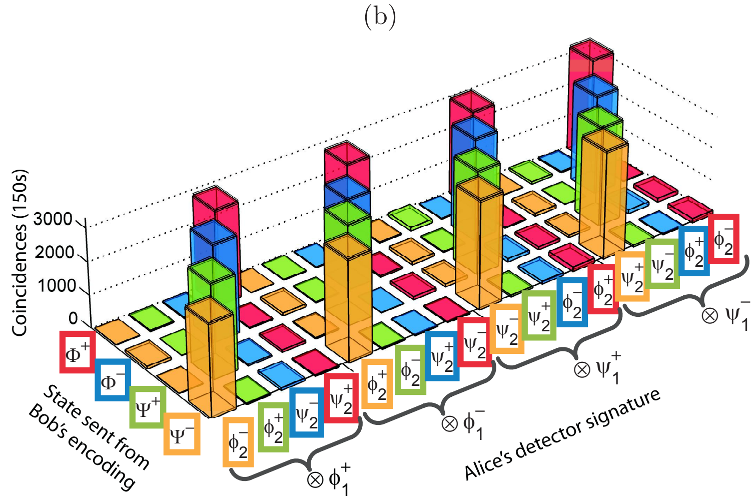
[¶]It should also be noted that two experiments in non-photonic systems have also implemented quantum dense coding. However, although these experiments in principle should be able to achieve a channel capacity of 2, the quality of the results limited the actual channel capacities to 1.16 (in ionic qubits[39]) and 1.3 (in an NMR system[40]).



(a)

State	Detector signature			
$\|\Phi^{\pm}\rangle$	$A_1^+ A_2^{\pm}$	$B_1^+ B_2^{\pm}$	$A_1^- A_2^{\mp}$	$B_1^- B_2^{\mp}$
$\|\Psi^{\pm}\rangle$	$A_1^+ B_2^{\pm}$	$B_1^+ A_2^{\pm}$	$A_1^- B_2^{\mp}$	$B_1^- A_2^{\mp}$

(b)



(c)

Figure 4. (a) System to enable full polarization Bell-state analysis and super-dense coding using localized measurements. Hyperentangled photons are directed into two polarization-spatial interferometers. (b) Coincidence signature between the upper and lower interferometer indicates unambiguously the polarization Bell state of the two photons. (c) Experimental results of our super dense coding experiment. The rows show which polarization Bell state Bob prepared, while the columns indicate the results of Alice's measurements on the two photons.

operation between the two degrees of freedom, implemented using the hologram-polarizing beam splitter interferometer described earlier. The net result is that each of the four polarization Bell states results in a different coincidence signature for the two photons, as shown in Fig. 4(b). One set of results is shown in Fig. 4(c). We've been able to achieve thus far an average success probability for determining the encoded Bell state of 95%, corresponding to a channel capacity of 1.630 (6). Table 1 shows an error budget for our experiment, indicating the imperfections we have identified and their reductions to the expected channel capacity. After accounting for these errors, the net theoretical channel capacity is 1.64 (2), in good agreement with our experimental results.

Table 1. Superdense-coding error budget. The state imperfections affecting the channel capacity (CC) were inferred from complete state tomography.

Effect	Reduction to CC
Imperfect state	
$T_{pol} = 96.7(8)\%$, $S_{pol} = 2.0(4)\%$	-0.09(2)
$T_{OAM} = 91(3)\%$, $S_{OAM} = 6(2)\%$	-0.20(3)
Polarizing beam splitter crosstalk	
$T_V = 1.0(2)\%$, $R_H = 0.5(1)\%$	-0.10(2)
Accidental coincidences	
5 in 150 s	-0.02
Total reduction	-0.36(5)
$CC_{exp} = 1.630(6)$ *vs*	$CC_{theory} = 1.64(2)$

One might question whether it is legitimate to call the previous scheme super-dense coding, given that the introduction of an extra degree of freedom increases the overall size of the Hilbert space to 16, i.e., there are 16 hyper-Bell states of the form $|\chi_{polarization}\rangle \otimes |\chi_{spatial}\rangle$ which span the total Hilbert space. Because Bob actually has access to two qubits on his photon, it is perhaps not surprising that he is able to use them to send two classical bits of information to Alice. However, we would like to stress that in applying his unitary transformations, *Bob only ever changes the polarization state of his photon* before sending it on to Alice. Thus, the presence of the auxiliary spatial mode qubit is in some sense irrelevant here. To put it differently, one could imagine a different situation where the two qubits were actually separable, and that Alice held onto the auxiliary qubit, and only the polarization qubit went to Bob. In this case, he would still be able to use the hyperentanglement-enhanced techniques to send two full classical bits to Alice using his single-qubit photon.

One might also ask whether or not one could employ the full structure of the Hilbert space to enable 16 messages to be sent, i.e., to achieve a channel capacity of 4. Unfortunately, we have shown that with linear optics this is not possible;[42] in particular, we have proved that one can at most reliably distinguish seven different subspaces (and only 6 unless one has photon-number-resolving detectors), leading to a maximum channel capacity of 2.8 bits per photon. One experimental scheme to realize this is presented in 36, 37, and discussed further in Ref.42.

In conclusion, we have seen that the use of hyperentanglement enables a number of otherwise impossible quantum communication protocols, and enhances the capabilities of others. Via the process of parametric downconversion in a double-crystal scheme, we have demonstrated high quality hyperentanglement of photon pairs. These have enabled us to remotely prepare single-photon two-degree-of-freedom entangled states, to realize complete polarization Bell-state analysis with only linear optics, and to exceed the threshold for channel capacity in a super-dense coding experiment restricted to linear optics. We are enthusiastic about further applications of hyperentanglement to other problems in quantum communication and quantum computing.

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