1. Sierpinski carpet is produced by removing from a square its central part with the size $1/3$ of the square edge. Then the procedure is repeated with each of the 8 remaining parts, and so on ad infinitum. Find the fractal dimension of the carpet.

2. Density of percolation clusters of mass $S$ has the following functional form: $N_S \propto S^{-\tau} \exp\left(-S/S_{\text{max}}\right)$, where

\[ \tau = 1 + \frac{d}{d_c} \]

is the power-law exponent, $d_c < d$ is the fractal dimension of the $\infty$-cluster, $S_{\text{max}} = \bar{S} d_c$ is the mass of the typical critical cluster, $\bar{S} = |x-x_c|^{-\nu}$ is the diameter of such a cluster, and $\nu$ is another exponent.

Other standard critical exponents $\alpha, \beta, \gamma$ are defined by:

\[ \bar{S} \sum_{S_{\infty}} N_S \sim |x-x_c|^{-d} \]
\[ \bar{S} \sum_{S_{\infty}} S N_S \sim |x-x_c|^\alpha \]
\[ \bar{S} \sum_{S_{\infty}} S^2 N_S \sim |x-x_c|^\beta \]

Here $\bar{S}$ means "singular" (non-analytic) part. Prove the following relations:

\[ \alpha = 2 - \nu d \]
\[ \beta = \nu \cdot (d - d_c) \]
\[ \gamma = \nu d - 2\beta \]

3. Suppose that the density of states has a power-law suppression near the Fermi energy ($E \approx 0$):

\[ \psi(E) \propto |E|^\mu \]

$\mu > 0$. Adapt Mott's argument for variable-range hopping to this case and show that $R \sim \exp\left[\left(\frac{1}{\gamma}\right)^\xi\right]$, where

\[ \xi = \frac{\mu + 1}{\alpha + \mu + 1} \]

In particular, if $\mu = d - 1$, which can be due to electron interactions (Coulomb gap), then $\xi = 1/2$ - the Efros-Shklovskii law.