Physics 120A
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Due: At the start of lab section, the week of April 19, 2004

Experiment #2: DC and AC Wheatstone Bridges

Purpose:
Building on the initial work that was done last week, the purpose of this lab is to demonstrate the use of a Wheatstone bridge to perform a resistance measurement, and to introduce you to thermistors. Also, while a Wheatstone bridge cannot be balanced in the presence of a single reactive element, it can still be utilized to measure a single reactive component. To do this measurement requires use of some of the available features of the oscilloscope, which you will learn in this lab.

Equipment:
This lab will require the following items:

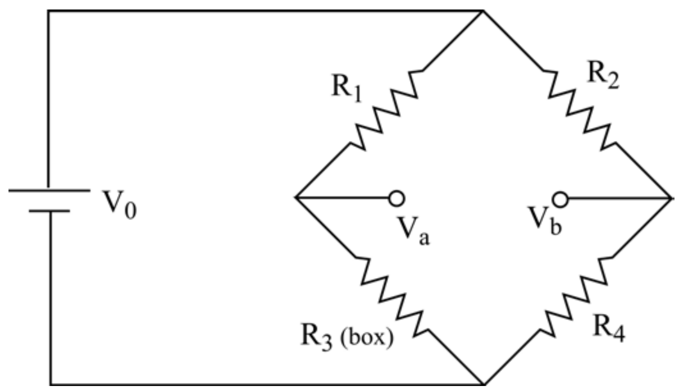
- Oscilloscope
- Digital multimeter
- Appropriate clip-leads and banana cables
- Power supply
- Function generator
- Resistor "decade" box
- Three 1 kΩ resistors
- A 0.01 μF capacitor
- An NTC thermistor
- The Extech 40131K electronic thermometer
- A thermal block (aluminum) with 100 Ω heating resistor

NOTE: The following is repeated from last week's lab for convenience:

The DC Bridge Circuit
Bridge circuits provide a sensitive means of comparing electrical signals or components. They were first developed by Wheatstone to compare resistances.

In Fortney, p.12, a bridge circuit is analyzed, using the branch method. The Thévenin analysis of the same circuit (Fortney, p. 18) allows us to study the effect of the finite input resistance of the **null detector** (our DMM, or oscilloscope). In this experiment we will:

Set up a DC Wheatstone bridge to study the measurement of resistances and the resolution of these measurements.



DC Wheatstone Bridge: Theory
The analysis in Fortney, p.18 considers the Wheatstone bridge as a voltage supply (terminals **a** and **b**, with a Thevenin voltage V_{Th} and a Thévenin resistance R_{Th}.

$$V_{Th} = V_{ab,open} = V_0\left(\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4}\right)$$

$$R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

If we choose all resistances to be 1 kΩ, then R_{Th} = 1 kΩ; thus, if V_{ab} is measured with a DMM (input resistance = 10 MΩ), our measurement will be erroneously low by only 0.01 %.

To *balance the bridge* means to adjust R_3, the decade resistor box, so that V_{ab} = 0 V. Under this condition

$$R_1 = R_3 \frac{R_2}{R_4}$$

DC Wheatstone Bridge: Experiment

If you had difficulty last week in setting up the DC bridge, quickly run through these steps just to make certain you fully understand how to achieve balance. *It is not necessary to write this section up again!*

1. Select three 1 kΩ resistors (check the color code).

2. Measure and record their resistance, using the DMM.

3. Warm one resistor with your fingers, while measuring. Does the resistance measurement change noticeably?

4. Using your "proto" board, set up a bridge as shown in the figure above.

5. Set the DC power supply to 1 V and connect it to the bridge.

6. Connect the decade resistor box as shown and the DMM between points **a** and **b**.

7. Increase the sensitivity of the DMM until you get a non-zero reading.

8. Change the setting of the decade resistor box (Rbox) to reduce to zero the reading on the DMM.

9. Repeat the previous two steps until the bridge is balanced with the DMM on its most sensitive scale.

10. How well does this setting agree with above balance condition?

Introduction to Thermistors

The **thermistor** is a resistor whose resistance is **significantly** temperature-dependent, as follows:

$$\frac{R}{R_0} = e^{\alpha(T-T_0)}$$

where

$$\alpha = \frac{1}{R}\frac{dR}{dT}\bigg|_{T_0}$$

Thermistors come with positive or negative temperature coefficients α, and, hence, are called PTC thermistors or NTC thermistors. Both PTC and NTC thermistors are used for temperature sensing. PTC thermistors (resistance increases with temperature), when used in series with a voltage supply, help protect the supply against overload.

In this experiment we will study a temperature-sensing NTC thermistor. Standard commercial NTC thermistors have temperature coefficients in the range of a=2 to 5%/°C.

As with all resistors, current through the thermistor will heat it, whereas the ambient air will cool it down, giving an equilibrium temperature above the ambient temperature. The heat dissipation coefficient depends on the structure of the thermistor and, for standard commercial heat sensing thermistors, is about 6 mW/°C. That is, 6 mW of power dissipation will cause the resistor to be 1°C above ambient temperature.

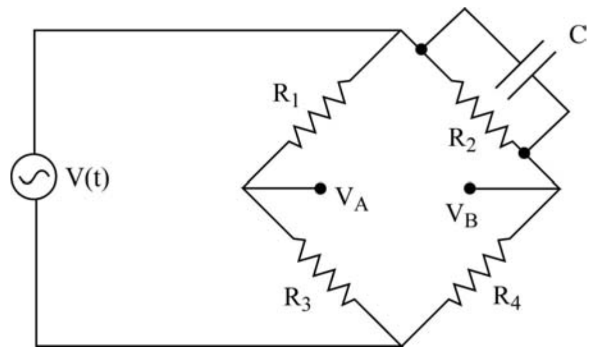
To measure temperatures, we use the Extech 40131K electronic thermometer. To insure that both the thermistor and the thermometer are at the same temperature, we locate each of them in its hole in an aluminum block (aluminum is a good thermal conductor). A 100 Ω, 3 watt resistor, mounted on the side of the aluminum block, will be used to raise its temperature.

Note: 3 watts is dissipated in a 100 Ω resistor when the current through it is 173 mA (or the voltage across is 17.3 V). If you exceed these limits, you will destroy the resistor.

1. Replace the resistor R4 in your bridge circuit by the thermistor.
2. Locate the thermistor and the electronic thermometer in the holes in the aluminum block.
3. Turn on the thermometer (push the POWER button), select the centigrade scale and the 0.1°C sensitivity.
4. When this system has stabilized at room temperature, balance the bridge (V_{AB}=0 V) with the decade resistor box. Record both the temperature and the R_{box} setting.
5. Using one section of your power supply, set the resistor dissipation to about 1 watt (10 V) and, when the temperature has stabilized, balance the bridge and record the box setting and the temperature.
6. Change the heating to several other settings (within the safe range) and repeat the above procedure.
7. Plot your data on a semilogarithmic graph ($\log[R/R_0]$ vs. T) and do a straight line fit. Do your data, within your measurement errors agree with the above formula? How did you determine your measurement errors?
8. To what extent did Joule heating from the current through the thermistor affect your measurements?

AC Wheatstone Bridge: Theory

If one of the bridge elements is reactive, i.e. not purely resistive, the complex impedance (see Fortney, p.54) must replace the corresponding resistance in the above treatment. Now $V_{AB} = V_A - V_B$ will have a magnitude and a phase. To balance the bridge, i.e. get $V_{AB}=0$, both the magnitudes and phases of V_A and V_B must be equal, i.e. an appropriate reactive element must be placed in another branch of the bridge.



Here we will study the effect of a capacitor in parallel with either R_2 or R_4 (denoted $R_{2,4}$). In this case, the impedance Z of the two in parallel is given by

$$\frac{1}{Z} = \frac{1}{R_{2,4}} + \frac{1}{Z_C} = \frac{1}{R_{2,4}} + j\omega C = \frac{1}{R_{2,4}}\left(1 + j\omega R_{2,4} C\right)$$

Consider the case where the capacitor is in parallel with R_2. We obtain

$$\frac{V}{V_B} = R + \frac{Z}{R_4} = 1 + \frac{1}{\alpha\left(1 + j\beta\right)} \rightarrow 1 + \frac{1}{\left(1 + j\beta\right)} = \frac{2+\beta}{1+\beta}$$

where, for convenience, we have defined

$$\alpha = \frac{R_4}{R_2} \rightarrow 1$$

$$\beta = \omega R_2 C = \frac{R_2}{\left|Z_C\right|}$$

This yields

$$\frac{V_B}{V} = \frac{\alpha\left(1+\beta^2\right)}{\left(1+\alpha+\alpha\beta^2\right) - j\beta} \rightarrow \frac{\left(1+\beta^2\right)}{\left(2+\beta^2\right) - j\beta}$$

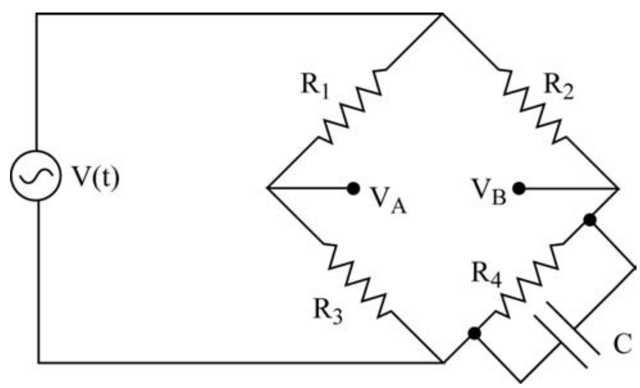
For R_2=R_4,

$$\frac{V_B}{V} \to \frac{1+\beta^2}{\sqrt{4+5\beta^2+\beta^4}} = \sqrt{\frac{1+\beta^2}{4+\beta^2}}$$

$$\tan(\varphi) \to \frac{\beta}{2+\beta^2}.$$

The phase angle φ has a maximum deviation from zero at that frequency ω where $\beta^2 = 2$. This gives

$$\varphi = 0.340, \quad \omega = \frac{\sqrt{2}}{R_2 C} \quad and \quad \frac{|V_B|}{|V|} = \frac{1}{\sqrt{2}} = 0.707\,.$$



In the case where the capacitor is in parallel with R_4, we have

$$\frac{V}{V_B} = 1 + \frac{1}{\alpha}(1+j\alpha\beta) = \left(1+\frac{1}{\alpha}\right) + j\beta$$

Again, with R_2=R_4, α=1, we get

$$\frac{V}{V_B} \to 2 + j\beta$$

or

$$\frac{|V|}{|V_B|} \to \sqrt{4+\beta^2}$$

$$\tan(\varphi) \to \frac{\beta}{2}$$

Here the phase angle φ increases continuously to π/2 at very large frequencies. At φ=π/4, β=2 we have

$$\frac{|V_B|}{|V|} \to \frac{1}{\sqrt{8}} = 0.353\,.$$

The AC Wheatstone Bridge: Experiment

[Note: the oscilloscopes this year are new and may not have the exact same functionality as indicated below. Try to achieve the same results as indicated, but make sure to ask the TA if you have any questions].

In the case of the DC Wheatstone bridge, both the power supply and the null measuring instrument (the DMM) are *floating*; that is none of their terminals have internal connections to the outside world. For the AC Wheatstone bridge, the audio oscillator has the **black terminal connected to ground**. The null measuring instrument is your oscilloscope, which also has the shell of its BNC input connector connected to ground. With the junction between R_3 and R_4 connected to the ground (black terminal) of the audio oscillator (as in the figure above), we cannot use the oscilloscope to directly measure the signal V_A - V_B because the scope would ground point B! Instead we will measure the signal V_A at the junction between R_1 and R_3 with respect to ground on one oscilloscope channel, and the signal V_B at the junction between R_2 and R_4 with respect to ground on the other channel.

1. To measure the amplitude ratio and phase difference, we use one of the two methods described at the end of this chapter. **Read these now**. The direct measurement of A, B, T and Δ, is appropriate in most cases. For maximum precision, do the time measurements at the "zero crossing" points of the sine waves. The A-B method allows you to measure φ to higher precision when φ << 1 and A and B differ by no more than a factor of two or three.

2. Using coaxial cables with clip leads at one end, connect the audio oscillator between V and ground, channel 1 of the oscilloscope between V_A and ground, and channel 2 of the oscilloscope between V_B and ground.

3. The capacitor which, eventually, you will connect in parallel with $R_{2,4}$ is 0.01 μF. Compute the frequency ω_m at which the phase of V_B, when connected across R_2, is a maximum.

4. Set the audio oscillator to that frequency, with an amplitude of about 1 V, and balance the bridge, using the resistor box. By superimposing the V_A and V_B traces, confirm that, in this balance condition the two traces are indeed identical in amplitude and phase. Use the (A-B) method to check for small phase differences. Does the bridge remain in balance at high and low frequencies?

5. Put the bridge, using the resistor box, **off balance** by a small amount. Performing repeated measurements (done separately by yourself and your partner to avoid biases) determine your measurement errors in phase and amplitudes.

6. Now we are ready to do reactance measurements and check them against our theory. Balance the bridge. Connect a 0.01 μF capacitor in parallel with R_2 and measure and record, over a wide range of frequencies, centered at ω_m, the amplitude ratio and phase difference between the signals at points A and B. Next, connect a 0.01 μF capacitor in parallel with R_4, and repeat the measurements. Don't forget to measure the frequency. How well does it agree with the dial setting?

7. Make a plot of the "expected" amplitude ratios and phase differences, as a function of frequency, for both capacitor configurations. On these plots, record the above measurements, including their measurement errors, derived from the above error measurements. How well do your measurements agree with your predictions? If there are significant differences, try to explain their source.

Methods to Measure the Transfer Function

We frequently wish to measure an output signal as a function of the corresponding input signal, i.e., a *transfer function* (see Fortney, p.71). If the input and output signals at a given frequency ω are

$$\tilde{v}_i(t) = |\tilde{v}_i| e^{j\omega t} \quad \text{and} \quad \tilde{v}_o(t) = |\tilde{v}_o| e^{j(\omega t+\varphi)}$$

then the transfer function is

$$\tilde{H}(\omega) = \frac{\tilde{v}_o}{\tilde{v}_i} = \frac{|\tilde{v}_o|}{|\tilde{v}_i|} e^{j\varphi}$$

where φ(ω) is the *phase shift* of the output signal with respect to the input signal. Thus, we want to measure the **ratio** of amplitudes and the **difference** between the phases. Your oscilloscope, the Tektronix TAS455, has the following features:

1. Signals on two inputs can be displayed simultaneously.
2. Both time and amplitude axes are linear over the whole screen.
3. The time scale is given by a precision oscillator. The relative amplitude scale is given by precision attenuators.
4. You can display the sum or difference of the two input signals.
5. Using the amplitude and time *fiducials*, we can accurately measure amplitudes and time differences.

This allows us to determine amplitude ratios and phase differences in two different ways:

Direct Method: A, B, T, Δ

1. Display the two sinusoidal signals.
2. Measure the peak-to-peak height of each signal (called A and B), using the amplitude fiducials. The amplitude ratio B/A is the magnitude of the transfer function.
3. Measure the period T of one signal using the time fiducials.
4. Measure the time difference Δ between two points of equal phase on the two signals using the time fiducials. The phase difference is j=2πΔ/T.

A-B Method

Display the two sinusoidal signals, and **INVERT** signal B. Using the **ADD** function of the ALT/CHOP ADD menu, display the difference between the two signals. The difference has amplitude R and phase δ given by

$$R\cos(\omega t+\delta) = A\cos(\omega t) - B\cos(\omega t+\varphi)$$

$$\left(\frac{R}{A}\right)^2 = 1+\left(\frac{B}{A}\right)^2 - 2\frac{B}{A}\cos(\varphi)$$

$$\frac{A}{B} = \cos(\varphi)+\frac{\sin(\varphi)}{\tan(\delta)}$$

1. Measure the amplitude A of the first signal.

2. Measure the amplitude R of the difference signal and the phase difference δ between A and R.

This method is useful when the phase difference between A and B is small (φ << 1) and A~B, i.e., when the transfer function is close to 1 and close to real. In that case

$$R \sim A - B$$

$$\varphi \sim \frac{R/A}{1-R/A}\tan(\delta) \sim (A/B-1)\tan(\delta)$$

Thus, R measures directly (A-B). The measured tan(δ) is proportional to φ.