• Reminder:
  – Bjorken Scaling
• QCD and Scaling violations
• Reminder:
  – PDF
  – Parton-Parton luminosities
• Outlook on this Quarter
Logic of what we did:

• Electron - muon scattering in lab frame
  – Show what spin 1/2 on spin 1/2 scattering looks like for point particles.

• Elastic electron - proton scattering
  – Introduce the concept of form factors
  – Show how the charge radius of proton is determined

• Inelastic electron - proton scattering
  – Parameterize cross section instead of amplitude

• Deep inelastic electron - proton scattering
  – Discuss Bjorken scaling
  – Introduce partons and parton density function
  – Discuss parton density function of proton

• Construct “parton-parton luminosity” for pp and ppbar
  – Explain the excitement about the LHC
Cross section for inelastic electron proton scattering

\[
\left. \frac{d\sigma}{d\Omega} \right|_{\text{lab}} = \frac{4\alpha^2}{4\,E^2\sin^4\frac{\theta}{2}} \left[ W_2 \cos^2\frac{\theta}{2} + 2W_1 \sin^2\frac{\theta}{2} \right]
\]

\[W_i = W_i(\nu,q^2)\]

- \(p\) = incoming proton 4-momentum
- \(k\) = incoming electron 4-momentum
- \(k'\) = outgoing electron 4-momentum
- \(m\) = proton mass

\[q^2 = (k - k')^\mu (k - k')_{\mu}\]
\[\nu \equiv \frac{p \cdot q}{m}\]

See H&M chapter 8.3 for details.
Deep inelastic scattering

- Intuitively, it seems obvious that small wavelength, i.e. large \(-q^2\), virtual photons ought to be able to probe the charge distribution inside the proton.
- If there are pointlike spin 1/2 particles, i.e. “quarks” inside, then we ought to be able to measure their charge via electron-proton scattering at large \(-q^2\).
- Within the formalism so far, this means that we measure \(W_1\) and \(W_2\) to have a form that indicates pointlike spin 1/2 particles.

What’s that form?

Let’s compare e-mu, and deep inelastic scattering.
**Electron muon:**

\[
\left. \frac{d\sigma}{dE'd\Omega} \right|_{lab} = \frac{4\alpha^2 E'^2}{q^4} \left[ \cos^2 \frac{\theta}{2} - \frac{q^2}{2m^2} \sin^2 \frac{\theta}{2} \right] \delta \left( \nu + \frac{q^2}{2m} \right)
\]

**Inelastic electron proton:**

\[
\left. \frac{d\sigma}{dE'd\Omega} \right|_{lab} = \frac{4\alpha^2 E'^2}{q^4} \left[ W_2 \cos^2 \frac{\theta}{2} + 2W_1 \sin^2 \frac{\theta}{2} \right]
\]

From this we concluded that \( W_1 \) and \( W_2 \) for a point particle are given by:

\[
W_1 = -(q^2/2M^2) \delta[\nu+(q^2/2m)]
\]

\[
W_2 = \delta[\nu+(q^2/2m)]
\]
Rewriting and simplifying:

• Let’s replace \(-q^2\) by \(Q^2\) in order to always have a positive \(Q^2\) value in all our expressions.

• Mathematical aside: \(\delta[ax] = \delta[x]/a\)

• Notational aside:

\[
x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2mv}
\]

\(x\) is a dimensionless quantity that is interpreted as the longitudinal momentum fraction of the parton inside the proton.
$W_1, W_2$ for point particles in proton

$$2mW_1 = \frac{Q^2}{2m} \delta \left( \nu - \frac{Q^2}{2m} \right) \Rightarrow 2mW_1 = \frac{Q^2}{2m\nu} \delta \left( 1 - \frac{Q^2}{2m\nu} \right)$$

$$\nu W_2 = \delta \left( 1 - \frac{Q^2}{2m\nu} \right)$$

Both of these structure functions are now functions of only one dimensionless variable, $x$ !!!

$\Rightarrow$ Bjorken Scaling

Both of these structure functions are obviously related. In this case, there is only one $F(x)$. 
Scaling as characteristic of point particles inside the proton

- To understand why the scale independence itself is the important characteristics of having point particles inside the proton, compare $W_i$ for e-mu with elastic e-proton:

\[
2mW_1 = \frac{Q^2}{2mv} \delta \left(1 - \frac{Q^2}{2mv}\right) \quad 2MW_1^{\text{elastic}} = G(Q^2) \frac{Q^2}{2Mv} \delta \left(1 - \frac{Q^2}{2Mv}\right)
\]

\[
\nu W_2 = \delta \left(1 - \frac{Q^2}{2mv}\right) \quad \nu W_2^{\text{elastic}} = G(Q^2) \delta \left(1 - \frac{Q^2}{2Mv}\right)
\]

- For elastic scattering, there is an explicit $Q^2$ dependence. The 0.71GeV mass scale in the pole of $G$ sets a size cut-off below which the proton is more likely to disintegrate than scatter elastically.

\[
G = \frac{1}{\left(1 + \frac{Q^2}{0.71}\right)^2}
\]
Bjorken Scaling

$$\omega = \frac{2q \cdot p}{Q^2}$$

Fig. 15.9. Scaling behaviour of electromagnetic structure function $\nu W_2$ at various $\omega$ values. There is virtually no variation with $Q^2$. (From Panofsky, 1968.)
Recap so far

• We saw that point particles inside the proton lead to bjorken scaling
  – Historically this was essential to convince humanity that quarks exist in nature, and aren’t just some group theory tool to explain spectroscopy.

• However, the proton isn’t just non-interacting point particles with empty space in between.
  – We will now discuss the impact of QCD on the parton model “qualitatively”
  – See H&M chapter 10 for a more quantitative discussion.
Lowest order QCD corrections to “naïve” Parton Model

These diagrams have two experimentally visible consequences:

1. Outgoing quark no longer collinear with virtual photon. Or for hadron colliders, COM of “hard collision” no longer has zero pT.
2. Scaling violation of the proton structure function.
PT boost at Tevatron and LHC

• The primary source of transverse momentum of DY, Higgs production via gg, diboson production, etc. etc. etc. is gluon radiation.
Example: Higgs production

The differences are in the model used to "parameterize" higher order effects.

All distributions are normalized to the same area.
Scaling Violation

- As $Q^2$ increases, we probe shorter distances.

- At large $Q^2$ the large $x$ quarks are more likely to loose energy due to gluon radiation.
  - Increased quark content at low $x$ and high $Q^2$
  - Decreased quark content at large $x$ and high $Q^2$
Resulting Structure Functions

Low $x$

scaling at $x \sim 0.1-0.2$

High $x$
$Q^2$ dependence is moderate except for very small $x$ and/or very small $Q^2$. 

Example: Gluon PDF
Example: Up from valence & sea

Again, differences are large for small $x$ and small $Q^2$. 
Sources of info on PDF’s

Q (GeV)

10^{-1}

10^{0}

10^{1}

10^{2}

10^{3}

10^{4}

1/X

DIS (fixed target)
HERA (’94)
DY
W-asymmetry
Direct-γ
Jets
To accurately predict pdf’s for the relevant kinematics, we depend on QCD evolution of the structure functions.

So how well do we predict Parton-parton lumis at LHC?
The larger $x$ becomes, the larger the uncertainties.
Topics we omitted
and where to learn about them.

• Our treatment of QCD was by and large pathetic.
  – More on QCD improved parton model
    • Chapter 10 of H&M.
  – More on QCD & collider physics, especially LHC.
  – More on pdf’s and what data they come from
    • Atlas paper by Joey Huston (see course website).
  – More on a large variety of topics in QCD and collider physics.
    • Ellis, Stirling, Webber: QCD and Collider Physics