Lab Project / Homework III.

due June 1, 2001

Consider the unsteady 1D convection/diffusion equation with Dirichlet boundary conditions at both ends (ϕ is the temperature field):

at $x = 0$ $\quad \phi = 0$

at $x = 1$ $\quad \phi = 1$, $\quad L = 1$

At $t = 0$ $\quad \phi_0 = 0$ in the $[0, 1]$ interval, at any later time we have the Dirichlet boundary condition.

Choose $\Gamma = 0.1$ in the numerical solution, and $\rho = 1$, $u = 1$.

(1) With 41 nodes ($\Delta x = 0.025$) calculate the time step size where the explicit Euler method becomes unstable. Demonstrate the instability numerically, and explain what goes wrong physically.

(2) With the same uniform grid, consider the exact solution at $x = 0.95$ at time $t = 0.01$. Use the Crank-Nicolson scheme with $\Delta t = 0.0001$ (100 time steps) to obtain the "exact" solution at $t = 0.01$

Plot $\phi(x=0.95)$ as the number of steps used between $t=0$ and $t=0.01$ in the Crank-Nicolson, Implicit 3 Level, Implicit Euler, and Explicit Euler schemes for an accuracy test of the methods as a function of step size Δt. Vary Δt from $\frac{0.01}{5}$ to $\frac{0.01}{50}$ (5 to 50 steps) in the calculation.

(3) Change to $\Gamma = 0.02$ and calculate the heat flux across the mid point at $x=0.5$ as a function of time (every other parameter is the same as before)