Homework Set #1. due: April 23

Consider the Harmonic Oscillator

$$L(x, \dot{x}) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}x^2$$

under the following discretization procedure of the Feynman Path integral (Lecture 2):

$T_0 = 2\pi$ classical period of oscillator

$$\Delta t = \frac{T_0}{128}$$

$N_D = 600 \qquad x_0 = -4\,, \quad x_D = +4$

$x_{start} = 0.75$

$$\psi_0 = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha}{2}(x - x_{start})^2}$$ initial wavefunction

$\alpha = 2$

1. Calculate the propagator K from the elementary K_ε matrix $(N_D+1)\times(N_D+1)$ dimensional,

$\varepsilon = \frac{T_0}{128} = \Delta t$ for time period $\frac{T_0}{16}$

$$K = (\Delta x)^{N-1} \cdot K_\varepsilon^N(\Delta t)$$

2. Evolve the wavefunction in time with $\frac{T_0}{16}$ stepsize and measure $\langle x \rangle$ as a function of time. Make a plot

3. Calculate $\langle E \rangle$, $\langle K \rangle$, $\langle V \rangle$ as a function of time. Make a plot.

4. Calculate the evolution of the wavefunction as a function of time. Make plot.

5. Compare your plots with the first three plots of Lecture 2

6. Bonus : Animation of the wavefunction