Inductance $L$ is a measure of the self-induced emf

The self-induced emf is

$$\varepsilon = -N \frac{\Delta \Phi_B}{\Delta t}$$

but

$$\frac{\Delta \Phi_B}{\Delta t} \propto \frac{\Delta l}{\Delta t}$$

proportionality constant is $L$

$$\varepsilon = -L \frac{\Delta l}{\Delta t}$$

$L$ is a property of the coil, Units of $L$, Henry (H) $\frac{Vs}{A}$
Inductance of a solenoid with $N$ turns and length $\ell$, wound around an air core (assume the length is much larger than the diameter).

\[ \Delta B \quad \Delta I \]

\[ \frac{\Delta B}{\Delta t} \]

\[ \frac{\Delta l}{\Delta t} \]

\[ \Phi_B = BA = \mu_0 \frac{N}{\ell} I A \]

\[ \frac{\Delta \Phi_B}{\Delta t} = \mu_0 \frac{N}{\ell} \frac{\Delta I}{\Delta t} A \]

\[ \varepsilon = -N \frac{\Delta \Phi_B}{\Delta t} = -\mu_0 \frac{N^2}{\ell} A \frac{\Delta I}{\Delta t} = -L \frac{\Delta I}{\Delta t} \]

\[ L = \mu_0 \frac{N^2}{\ell} A \]

inductance proportional to $N$ squared x area/length
An air wound solenoid of 100 turns has a length of 10 cm and a diameter of 1 cm. Find the inductance of the coil.

\[ L = \mu_o \frac{N^2}{\ell} A = \mu_o \frac{N^2}{\ell} \pi \frac{d^2}{4} \]

\[ L = \frac{4\pi 10^{-7} (100)^2 \pi (0.01)^2}{0.1(4)} = 1.0 \times 10^{-5} H \]
The inductor prevents the rapid buildup of current

\[ \varepsilon = -L \frac{\Delta l}{\Delta t} \]

But at long time does not reduce the current, \( \frac{\Delta l}{\Delta t} = 0 \)

at \( t=\infty \)

\[ I = I_0 (1 - e^{-\frac{t}{\tau}}) \]

\[ \tau = \frac{L}{R} \]
Inductive reactance, $X_L$

$$\Delta V_L = X_L I$$

$$X_L = 2\pi fL$$

$$I = \frac{\Delta V_L}{X_L}$$

Dimensional analysis

$$\tau = \frac{L}{R}$$

$$\frac{1}{\tau} = \omega = 2\pi f = \frac{R}{L}$$

$$R = 2\pi fL$$

$$X_L = 2\pi fL$$

So $X_L$ has units of ohms
Applications of Inductors

Reduce rapid changes of current in circuits

Produce high voltages in automobile ignition.
Energy is stored in a magnetic field of an inductor.

Work is done against ε to produce the B field.

This produces a change in the PE of the inductor

$$PE_L = \frac{1}{2}LI^2$$

This stored PE can be used to do work
21.1 RLC circuit

AC circuits
RLC circuit
Resonance
AC Circuits

• Current changes with time
• Current is both positive and negative
• Voltage (V) and Current (I) are not always “in phase” – when the voltage is a max the current may not be a max
• Only for a resistor are V & I always “in phase” (voltage max occurs when current is max).
AC circuits-Resistor

Household currents are alternating currents AC that vary with time. For circuits only involving resistors the only difference is that the average currents and voltages must be used, $I_{\text{rms}}, V_{\text{rms}}$. (rms –root mean square)
**AC circuit with capacitors, inductors and resistor.**

Resistors, capacitors and inductors react differently to time dependent voltages.

These components behave differently in an AC circuit.

We have seen this already for the capacitor, so we already know about R & C, just need to examine L.
If the average voltages, currents and power are used then the relations for the between current, voltage and power are the same as for DC

\[ V_{\text{rms}} = I_{\text{rms}} R \]

\[ P_{\text{rms}} = I_{\text{rms}} V_{\text{rms}} = \frac{V_{\text{rms}}^2}{R} = I_{\text{rms}}^2 R \]
Response to a step voltage: Resistor, Capacitor, and Inductor

Capacitor blocks current at long times
Inductor blocks current at short times
Response to a sinusoidal voltage

V sinusoidal

I

VR

VC

VL

different phase shift between current and voltage

in phase with I

lags by 90°

leads by 90°
Response to a sinusoidal voltage

\[ V_{\text{rms}} = I_{\text{rms}} R \]

\[ V_{\text{rms}} = I_{\text{rms}} \left( \frac{1}{\omega C} \right) = I_{\text{rms}} X_C \]

\[ V_{\text{rms}} = I_{\text{rms}} (\omega L) = I_{\text{rms}} X_L \]

\[ X_C = \frac{1}{\omega C} \quad \text{Capacitive Reactance} \]

\[ X_L = \omega L \quad \text{Inductive Reactance} \]
Capacitive Reactance, $X_c$

$$\Delta V_C = X_c \cdot I$$

$$X_c = \frac{1}{\omega C}$$

$f=0$

DC

$I=0$

$f=\text{high}$

$I \text{ high}$

$X_c$ is higher at low frequency. The capacitor blocks current at long time, because more charge accumulates.

$X_c = \text{infinity}$

$X_c = \text{low}$
Capacitive Reactance

Dimensional analysis

\[ \tau = RC \]

\[ \frac{1}{\tau} = \omega = 2\pi f = \frac{1}{RC} \]

\[ R = \frac{1}{2\pi fC} \]

\[ X_c = \frac{1}{2\pi fC} \]

So \( X_c \) has units of Ohms
A 10 microfarad capacitor is in an ac circuit with a voltage source with RMS voltage of 10 V. a) Find the current for a frequency of 100 Hz. b) Find the current for a frequency of 1000 Hz.

a) \[ \Delta V_c = X_cl \]
\[ I = \frac{\Delta V_c}{X_c} = \frac{\Delta V_c(2\pi fC)}{1} \]
\[ I = 10(2\pi)(100)(10^{-5}) = 6.3 \times 10^{-2} \text{ A} \]

b) The frequency is 10 x higher, the current is 10 x higher
\[ I = 10 \times 6.3 \times 10^{-2} = 6.3 \times 10^{-1} \text{ A} \]
Inductive reactance, $X_L$

\[ \Delta V_L = X_L \Delta I \]

\[ I = \frac{\Delta V_L}{X_L} \]

\[ X_L = \omega L \]

An inductor has higher back emf when $\Delta I/\Delta t$ is greater, i.e. at high frequency. Inductive reactance higher at high frequency.
\[ X_L = 2\pi fL \quad I = \frac{\Delta V_L}{X_L} \]

Inductive reactance is higher at high frequency.
A inductor with $L = 10^{-5}$ H is driven by a 10 V ac source.

a) Find the current at $f = 100$ Hz.

b) Find the current at $f = 1000$ Hz

\[ I = \frac{10}{2\pi (100)(10^{-5})} = 1.6 \times 10^3 \text{ A} \]

b) the frequency is 10x greater
the current is inversely proportional to \( f \)
the current is 10x less

\[ I = \frac{1.6 \times 10^3}{10} = 1.6 \times 10^2 \text{ A} \]
Application.
High pass and low pass filters.

- Amplifier
- Capacitor
- Inductor
- Tweeter
- Woofer
- Stereo speakers

Pass high frequency/block low frequency
Pass low frequency/block high frequency
RLC circuit

- Currents and voltages are sinusoidal
- Charge and discharge of capacitor
- Energy only dissipated in R
- At resonance frequency maximum energy stored in electric and magnetic fields.
- This circuit can be used to pick out selected frequencies, e.g., in a radio receiver.
Voltage across $R$, $L$, $C$ are sinusoidal but with different phase relative to the current, and relative to each other.

The sum of voltages

$$\Delta V_s = \Delta V_R + \Delta V_L + \Delta V_C$$

But at any time the voltages are not maximum across $R$, $L$ and $C$ but differ because of phase shifts.
\[ I = i_{\text{max}} \sin \omega t \]

\[ \Delta V = \Delta V_R + \Delta V_L + \Delta V_C \]

\[ \Delta V = \Delta V_{\text{max}} \sin (\omega t + \Phi_i) \]

Sum of Voltages
Impedance, $Z$

\[ \Delta V = \sqrt{(\Delta V_L - \Delta V_C)^2 + \Delta V_R^2} \]

\[ \Delta V = \sqrt{(iX_L - iX_C)^2 + i^2R^2} \]

\[ \Delta V = i\sqrt{(X_L - X_C)^2 + R^2} \]

\[ \Delta V = iZ \quad \text{Like Ohm’s Law} \]

\[ Z = \sqrt{(X_L - X_C)^2 + R^2} \quad \text{L, C and R contribute to Z, Impedance.} \]
When $X_L = X_C$
then $X_L - X_C = 0$
$Z$ becomes a minimum
$I$ becomes maximum

If $R = 0$ then $I = \infty$ at resonance

resonance frequency

$$X_C = X_L$$

$$\frac{1}{2\pi f_0 C} = 2\pi f_0 L$$

$$f_0 = \frac{1}{2\pi \sqrt{LC}}$$
34. A resonance circuit in a radio receiver is tuned to a certain station when the inductor has a value of 0.20 mH and the capacitor has a value of 30 pF. Find the frequency of the station.

\[
 f_o = \frac{1}{2\pi \sqrt{LC}}
\]

\[
 f_o = \frac{1}{2\pi \sqrt{0.2 \times 10^{-3} \times (30 \times 10^{-12})}}
\]

\[
 f = 2.05 \times 10^6 \text{ Hz}
\]

\[
 f_o = 2.05 \text{ MHz}
\]
For LC circuit (R->0) at resonance
Energy oscillates between Electric and Magnetic Fields