

Relativity

Kinematics

- The gamma factor
- Length contraction
- Time Dilation
- Velocity Transformation
- Lorentz Transformation

Dynamics

- New definitions of energy and momentum
- Momentum conservation
- Energy conservation

QM Theory of Light

Blackbody Radiation

- Planck's Radiation Formula
- Stefan's Law, $P/A = \sigma(T^4)$
- Wien's Law, $\lambda(\text{peak}) = \text{constant} / T$

Photoelectric Effect

- The energy of a photon, $E = h \cdot f$

Compton Scattering

- Photons scattering from electrons
- Wavelength of scattered photons shifts because of energy conservation

Particle Nature of Matter

Thomson's Experiment

- Showed that there are light, negatively charged particles (electrons) within atoms

Milliken's Oil Drop Experiment

- Measured the charge of the electron accurately

Rutherford Scattering

- Established the existence of a small, heavy, positively charged atomic nucleus

The Bohr Model

- Explains the sharp spectral lines of hydrogen
- Uses QM to justify the existence of distinct energy levels
- Spectral lines are photons emitted or absorbed when an electron jumps between energy levels
- The Bohr Radius, $a_0 = \hbar^2 / (m_e \cdot k \cdot e^2) = 0.529 \text{ \AA}$

Matters and Waves

deBroglie Waves

- Assigns a wavelength to all particles related to its momentum, $\lambda = h/p$
- Justifies the existence of Bohr's assumed quantum energy levels

Double Slit Experiment

Standard experiment to test the existence of wave properties

When particles were shot through the slits, an interference pattern was observed, therefore the particles must have wave-like properties

Observe interference fringe maxima (constructive interference)

when $D \cdot \sin(\theta) = \lambda/2$, where D = separation of the slits

Heisenberg Uncertainty Principle

We can never measure both the momentum and position of a particle perfectly

$\Delta x \cdot \Delta p \sim h$, so if $\Delta p \rightarrow 0$, then $\Delta x \rightarrow \infty$

Same principle applies to energy and time, $\Delta E \cdot \Delta t \sim h$

QM in 1D

Rules of the Game

Given the potential field $U(x)$ (i.e. the force that's acting on the

particle $F = -dU/dx$), solve the time-independent Schrodinger

Equation to find the wave function and the allowed energy levels.

The absolute value of the square of the wave function $|\Psi(x)|^2$

represents the probability density of locating the particle at some

location x (The Born Interpretation). This imposes the

normalization requirement on the wave function, $\int |\Psi(x)|^2$

$dx = 1$

Other observable quantities are represented by Operators, and we

can calculate the values of the observable by operating on the

wave function with the corresponding operator.

Eigenfunctions of an operator are functions which satisfies the

equation $[Q] f(x) = Q \cdot f(x)$, where $[Q]$ is an operator, and Q is some

constant which we call the Eigenvalue

Eigenstates are quantum states whose wave functions are

eigenfunctions

If a state is an eigenstate of an observable operator $[Q]$, then the

observed value of $[Q]$ will always be the eigenvalue Q , i.e. no

uncertainty

If a state is not an eigenstate of $[Q]$, then each measurement of

Q will yield a different value. The average expected value is given

by $\int [Q]^* |\Psi|^2 [Q]$

The uncertainty in Q is given by $\Delta Q = \sqrt{\langle Q^2 \rangle - \langle Q \rangle^2}$, where

$\langle Q^2 \rangle$ is the expectation value of the $[Q]^2$ operator

Free particle, $U(x) = \text{constant}$

No forces acting on the particle, so it will propagate forever in either direction

$\Psi(x,t) = A \cdot \exp[i \cdot (kx - \omega t)]$

$k = p/\hbar$, $\omega = E/\hbar$

Particle in a Box, $U(x)=0$ between $0<x<L$, $U=\infty$ everywhere else

The particle acts like a free particle inside the box ($0<x<L$), but will never be outside the box, i.e. $\Psi = 0$ outside the box
 $\psi(x=0) = \psi(x=L) = 0$ imposes boundary conditions on our free particle wave function \Rightarrow Quantization of energy and wave function
 $\psi(x) = \sqrt{2/L} \cdot \sin(k \cdot x)$, $k = \sqrt{2 \cdot m \cdot E / (\hbar)^2} = n \cdot \pi \cdot x / L$
 $E_n = (\hbar)^2 \cdot k^2 / (2 \cdot m) = n^2 \cdot \pi^2 \cdot (\hbar)^2 / (2 \cdot m \cdot L^2)$ where $n=1,2,3,\dots$ is the quantum number labeling the state
The wave functions are sine waves (free particle wave functions) with nodes at the edges of the box (to satisfy the boundary conditions)

Finite Square Well, $U(x)=0$ between $0<x<L$, $U(x)=\text{constant}>E$ everywhere else

Since walls aren't infinitely high, Ψ doesn't have to be 0 at $x=0$ and $x=L$
But $U>E$ outside the box (classically forbidden), so the probability to find the particle outside of box should decay away
Inside the box, $\psi(x) = A \cdot \sin(k \cdot x)$
Outside the box, $\psi(x) = A \cdot \exp(a \cdot x)$ for $x<0$ or $\psi(x) = B \cdot \exp(a \cdot x)$ for $x>L$
 $a^2 = 2 \cdot m \cdot (U-E)$ outside the box
This changes the box energy levels slightly since wave function now penetrates outside the box by an amount $d = 1/a$
We can approximate the energy levels by slightly expanding our box by the penetration depth d , $E_n = n^2 \cdot \pi^2 \cdot (\hbar)^2 / [2 \cdot m \cdot (L+2 \cdot d)^2]$

Harmonic Oscillator, $U(x) = 1/2 \cdot K \cdot x^2 = 1/2 \cdot m \cdot \omega^2 \cdot x^2$

Has a characteristic frequency $\omega = \sqrt{K/m}$
Actual wave functions are complicated, but general shape is similar to box wave functions, i.e. ψ is wavelike, and the higher the n , the higher the energy, the more wiggles in the wave function.
But energy levels are real simple, $E_n = (n+1/2) \cdot \hbar \cdot \omega$
Note in this case $n = 0, 1, 2, \dots$, and the ground state is $n=0$, not $n=1$ like in box or in hydrogen atom, because $E(n=0)$ is the lowest *non-zero* energy level

Tunneling Phenomena

Recall Finite Square Well, where the wave function could penetrate slightly into classically forbidden regions ($U>E$)
Now consider a square barrier, $U(x) = \text{constant} > E$ between $0<x<L$ and $U=0$ everywhere else.
The particle with energy E will act like a free particle everywhere except in between $0<x<L$, where the wave function will decay exponentially in the

classically forbidden region

Classically, all particles will be reflected since $U > E$. But in QM, since wave function is non-zero in the barrier and on the other side, there is some probability that the particle can "tunnel" through the barrier and be found on the other side, with the exact same energy.

A number of incident particles will hit the barrier, some will be reflected, some will be transmitted.

Can calculate the reflection and transmission coefficients exactly by writing the wave function in each region, then solve for the normalizing coefficients by requiring that the wave function be continuous and smooth everywhere.

Look at the example of the potential step in the homework

For arbitrary potential shapes we can use an approximate formula

$T(E) \sim \exp[-2/\hbar \sqrt{2m} \int \sqrt{U-E} dx]$ where the integral is over the forbidden region only

Examples we've done: $U(x) = \text{constant}$, $U(x) = \text{triangle}$ (i.e. a line in $0 < x < L$)

General Ideas

The higher and the wider the barrier, the fewer particles will be transmitted

The total number of reflected and transmitted particles should equal the number of incident particles (i.e. $R+T = 1$)

QM in 3D

Now instead of just $U(x)$, we have $U(x,y,z)$ or $U(r, \theta, \phi)$

Idea is to solve the Schroedinger Eqn in each dimension separately. Each dimension will then have a corresponding quantum number associated with it.

3D Particle in a Box, $U(x) = 0$ inside $0 < x < L_x$, $0 < y < L_y$, $0 < z < L_z$

The three dimensions separate trivially into three 1D boxes

$\psi(x,y,z) = \psi(x) * \psi(y) * \psi(z)$, where $\psi(x)$ is the 1D particle in a box wave function with box size L_x , etc.

$E(n_x, n_y, n_z) = E(n_x) + E(n_y) + E(n_z)$ where $E(n_x)$ is the 1D box energy level

If $L_x=L_y=L_z$, then we have Degeneracy, where different combinations of (n_x, n_y, n_z) will give the same total energy.

Central Forces, $U(r)$ depends only on radial distance and not direction

The angular wave functions are always $Y_{lm}(\theta, \phi)$

Easy to read off the m quantum number from the Y_{lm} wave function, look for the $\exp(i*m*\phi)$ in the wave function (if there's no ϕ dependent factor, then $m=0$)

The quantum numbers l and m are related to the angular momentum, $L_z = m*\hbar$, $|L| = \sqrt{l*(l+1)} * \hbar$

Hydrogen and Hydrogen-like ions (one electron), $U(r) = -k * Z * e^2 / r$

It's a central force, so we know the angular wave functions already
The radial wave function $R(r)$ are complicated.

General ideas

$P(r) = r^2 * |R(r)|^2$ is the radial probability density (probability of finding the particle at r)

Most likely distance = maximum of $P(r)$

Average Distance = expectation value of $r = \int r * P(r) dr$

The energy levels we already know from Bohr's Model, $E_n = -k^2 e^2 / (2 * a_0) * (Z^2 / n^2)$, $E_n = -13.6 \text{ eV} (Z^2 / n^2)$

Quantum Number Rules

$n = 1, 2, 3, 4, \dots$ (Principal Quantum Number)

$l = 0, 1, \dots, n-1$ (Orbital Quantum Number)

$m_l = -l, \dots, l$ (Magnetic quantum number)

Atomic Structure

Notice that hydrogen energy levels only depend on n , so all (l, m_l) states are degenerate as long as they have same n

Other effects can break that degeneracy.

Zeeman Effect

The orbital motion and the spin of an electron both act like little current loops \Rightarrow magnetic moments for orbital and spin angular momentum

When external magnetic field is applied, the magnetic moments causes an energy difference between different angular momentum states

The total magnetic moment (including spin) is $u = u_B (m_l + g * m_s)$, where $g = 2$ for an electron

With a magnetic field B , the electron gains energy given by $U = u_B * B (m_l + g * m_s)$ where u_B is the Bohr magneton

Spin-Orbit Effect

In the electron frame, the nucleus is orbiting about the electron, which creates a magnetic field that affects the magnetic moment of the electron

Splits the (n, l, m_l, m_s) energy levels into different lines based on the *total* angular momentum J

$J = L + S$, $|J| = \sqrt{j * (j+1)} * \hbar$, $j = l + s, \dots, |l - s|$, $m_j = -j, \dots, j$

For an electron, splits each (l, s) level into two levels with $j = l + 1/2$ and $j = l - 1/2$

Multi-electron screening

Our calculations are all based on hydrogen-like systems, with one electron going around a nucleus with Z protons

For multiple electron systems, the other electrons in the way partly cancels the positive charge of the nucleus, so the outer electron only sees some $Z_{\text{eff}} < Z$ protons

Given the actual energy level of the outer electron, we define Z_{eff}

using the equation $E_n = -13.6 \text{ eV} (Z_{\text{eff}}^2 / n^2)$, i.e. pretend we're still a hydrogen atom, but with an effective nucleus instead of the actual nucleus with Z protons.

Periodic Table

Electrons are fermions and subject to the Pauli Exclusion Principle (only one particle per quantum state)

We add electrons to an atom starting at the lowest energy level, subject to Hund's Rule (fill equivalent orbitals with unpaired spins first), and Pauli Exclusion (one electron per n, l, m_l, m_s state).

Molecular Structure

It's often energetically favorable for multiple atoms to bond together.

Bonds happen because the combined energy of the electrons is lower in the bonded state than in their unbonded state

The equilibrium bond distance is the atomic separation which is the minimum for the bond potential, i.e. $dU/dr = 0$

The total electronic energy of two bonded electrons is $E(n_1) + E(n_2) + 2*(E_+)$ where $E(n_1)$, $E(n_2)$ are the standard hydrogen-like electron energies, and E_+ is the energy of the bonding orbital

In crystalline solids, many almost-degenerate hydrogen energy levels combine into energy bands.

The conducting properties of the solid depends on the location of Fermi energy level (the highest energy of the electrons given by Pauli Exclusion) vs. the energy bands

Useful Math

Integrals and Derivatives

Polynomials, r^n and r^{-n}

Square Roots

Sine and Cosine

Exponentials

Chain Rule

Integration by Parts

Algebra

Square Roots (solve for v given γ , square root of a sum is not equal to the sum of the square roots, etc)

Quadratic Formula

Imaginary numbers ($-i = 1/i$, $i^2 = -1$, etc)

Trigonometric Identities

$\tan(a) = \sin(a) / \cos(a)$

$\sin(a)^2 + \cos(a)^2 = 1$

$\sin(a+b) = \sin(a) \cos(b) + \sin(b) \cos(a)$

$\sin(a-b) = \sin(a) \cos(b) - \sin(b) \cos(a)$

$\cos(a+b) = \cos(a) \cos(b) - \sin(a) \sin(b)$

$\cos(a-b) = \cos(a) \cos(b) + \sin(a) \sin(b)$

Double angle formula = $\sin(a+a)$ or $\cos(a+a)$

Dimensional Analysis

Make sure your answers have the right units!