Physics 4D Winter 2012 Homework 3 Solutions

Chapter 32

5. The incoming ray is represented by line segment DA. For the first reflection at A the angles of incidence and reflection are θ_1. For the second reflection at B the angles of incidence and reflection are θ_2. We relate θ_1 and θ_2 to the angle at which the mirrors meet, ϕ, by using the sum of the angles of the triangle ABC.

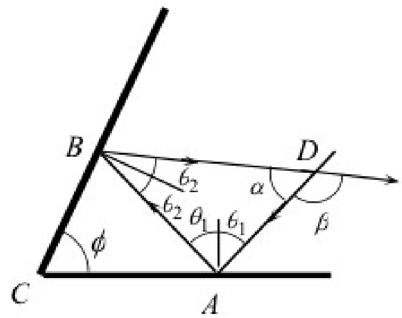
$$\phi + (90^\circ - \theta_1) + (90^\circ - \theta_2) = 180^\circ \quad \rightarrow \quad \phi = \theta_1 + \theta_2$$

Do the same for triangle ABD.

$$\alpha + 2\theta_1 + 2\theta_2 = 180^\circ \quad \rightarrow \quad \alpha = 180^\circ - 2(\theta_1 + \theta_2) = 180^\circ - 2\phi$$

At point D we see that the deflection is as follows.

$$\beta = 180^\circ - \alpha = 180^\circ - (180^\circ - 2\phi) = \boxed{2\phi}$$

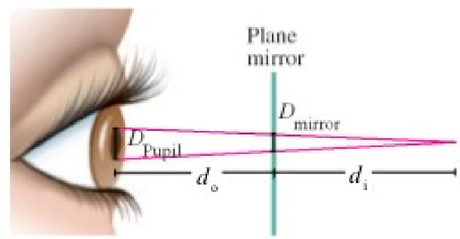


6. The rays entering your eye are diverging from the virtual image position behind the mirror. Thus the diameter of the area on the mirror and the diameter of your pupil must subtend the same angle from the image.

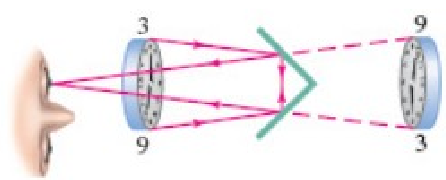
$$\frac{D_{\text{mirror}}}{d_i} = \frac{D_{\text{pupil}}}{(d_o + d_i)} \quad \rightarrow \quad D_{\text{mirror}} = D_{\text{pupil}} \frac{d_i}{(d_o + d_i)} = \tfrac{1}{2} D_{\text{pupil}}$$

$$A_{\text{mirror}} = \tfrac{1}{4}\pi D_{\text{mirror}}^2 = \tfrac{1}{4}\pi \tfrac{1}{4} D_{\text{pupil}}^2 = \frac{\pi}{16}(4.5\times10^{-3}\,\text{m})^2$$

$$= \boxed{4.0\times10^{-6}\,\text{m}^2}$$



7. See the "top view" ray diagram.



8. (*a*) The velocity of the incoming light wave is in the direction of the initial light wave. We can write this velocity in component form, where the three axes of our coordinate system are chosen to be perpendicular to the plane of each of the three mirrors. As the light reflects off any of the three mirrors, the component of the velocity perpendicular to that mirror reverses direction. The other two velocity components will remain unchanged. After the light has reflected off of each of the three mirrors, each of the three velocity components will be reversed and the light will be traveling directly back from where it came.

(*b*) If the mirrors are assumed to be large enough, the light can only reflect off two of the mirrors if the velocity component perpendicular to the third mirror is zero. Therefore, in this case the light is still reflected back directly to where it came.

11. The image flips at the focal point, which is half the radius of curvature. Thus the radius is $\boxed{1.0\text{ m}}$.

12. (*a*) The focal length is half the radius of curvature, so $f = \frac{1}{2}r = \frac{1}{2}(24\,\text{cm}) = \boxed{12\,\text{cm}}$.

(*b*) Use Eq. 32-2.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \;\rightarrow\; d_i = \frac{d_o f}{d_o - f} = \frac{(35\,\text{cm})(24\text{cm})}{35\,\text{cm} - 24\,\text{cm}} = \boxed{76\text{cm}}$$

(*c*) The image is $\boxed{\text{inverted}}$, since the magnification is negative.

15. The object distance of 2.00 cm and the magnification of +4.0 are used to find the image distance. The focal length and radius of curvature can then be found.

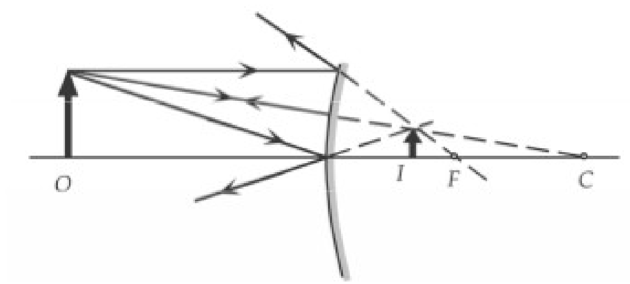
$$m = \frac{-d_i}{d_o} \;\rightarrow\; d_i = -md_o$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \;\rightarrow\; f = \frac{d_o d_i}{d_o + d_i} = \frac{d_o(-md_o)}{d_o - md_o} = \frac{md_o}{m-1} = \frac{4(2.00\,\text{cm})}{4-1} = 2.677\,\text{cm}$$

$$r = 2f = 2(2.667\,\text{cm}) = \boxed{5.3\text{cm}}$$

Because the focal length is positive, the mirror is $\boxed{\text{concave}}$.

18. (*a*) From the ray diagram it is seen that the image is virtual. We estimate the image distance as $\boxed{-6\,\text{cm.}}$



(*b*) Use a focal length of -9.0cm with the object distance of 18.0cm.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \;\rightarrow$$

$$d_i = \frac{d_o f}{d_o - f} = \frac{(18.0\,\text{cm})(-9.0\,\text{cm})}{18.0\,\text{cm} - (-9.0\,\text{cm})} = \boxed{-6.0\text{cm}}$$

(*c*) We find the image size from the magnification:

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o} \;\rightarrow\; h_i = h_o\left(\frac{-d_i}{d_o}\right) = (3.0\,\text{mm})\left(\frac{6.0\,\text{cm}}{18.0\text{cm}}\right) = \boxed{1.0\,\text{mm}}$$

21. Consider the ray that reflects from the center of the mirror, and note that $d_i < 0$.

$$\tan\theta = \frac{h_o}{d_o} = \frac{h_i}{-d_i} \;\rightarrow\; \frac{-d_i}{d_o} = \frac{h_i}{h_o}$$

$$m = \frac{h_i}{h_o} = \boxed{-\frac{d_i}{d_o}}$$