Physics 4D Winter 2012 Homework 1 Solutions

Chapter 31

1. The electric field between the plates is given by $E = \frac{V}{d}$, where d is the distance between the plates.

$$E = \frac{V}{d} \rightarrow \frac{dE}{dt} = \frac{1}{d}\frac{dV}{dt} = \left(\frac{1}{0.0011\,\text{m}}\right)(120\,\text{V/s}) = \boxed{1.1\times10^5\,\frac{\text{V/m}}{\text{s}}}$$

2. The displacement current is shown in section 31-1 to be $I_D = \varepsilon_0 A \frac{dE}{dt}$.

$$I_D = \varepsilon_0 A \frac{dE}{dt} = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(0.058\,\text{m})^2\left(2.0\times10^6\,\frac{\text{V}}{\text{m}\bullet\text{s}}\right) = \boxed{6.0\times10^{-8}\,\text{A}}$$

3. The current in the wires must also be the displacement current in the capacitor. Use the displacement current to find the rate at which the electric field is changing.

$$I_D = \varepsilon_0 A \frac{dE}{dt} \rightarrow \frac{dE}{dt} = \frac{I_D}{\varepsilon_0 A} = \frac{(2.8\,\text{A})}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(0.0160\,\text{m})^2} = \boxed{1.2\times10^{15}\,\frac{\text{V}}{\text{m}\bullet\text{s}}}$$

4. The current in the wires is the rate at which charge is accumulating on the plates and also is the displacement current in the capacitor. Because the location in question is outside the capacitor, use the expression for the magnetic field of a long wire.

$$B = \frac{\mu_0 I}{2\pi R} = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{R} = \frac{\left(10^{-7}\,\text{T}\bullet\text{m/A}\right)2\left(38.0\times10^{-3}\,\text{A}\right)}{(0.100\,\text{m})} = \boxed{7.60\times10^{-8}\,\text{T}}$$

After the capacitor is fully charged, all currents will be zero, so the magnetic field will be $\boxed{\text{zero}}$.

6. (*a*) The footnote on page 816 indicates that Kirchhoff's junction rule is valid at a capacitor plate, and so the conduction current is the same value as the displacement current. Thus the maximum conduction current is $\boxed{35\,\mu\text{A}}$.

(*b*) The charge on the pages is given by $Q = CV = C\mathscr{E}_0 \cos\omega t$. The current is the derivative of this.

$$I = \frac{dQ}{dt} = -\omega C\mathscr{E}_0 \sin\omega t \;;\; I_{max} = \omega C\mathscr{E}_0 \rightarrow$$

$$\mathscr{E}_0 = \frac{I_{max}}{\omega C} = \frac{I_{max} d}{2\pi f \varepsilon_0 A} = \frac{\left(35\times10^{-6}\,\text{A}\right)\left(1.6\times10^{-3}\,\text{m}\right)}{2\pi(76.0\,\text{Hz})\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\pi(0.025\,\text{m})^2}$$

$$= 6749\,\text{V} \approx \boxed{6700\,\text{V}}$$

(*c*) From Eq. 31-3, $I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$.

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt} \rightarrow \left(\frac{d\Phi_E}{dt}\right)_{max} = \frac{(I_D)_{max}}{\varepsilon_0} = \frac{35\times10^{-6}\,\text{A}}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{4.0\times10^6\,\text{V}\bullet\text{m/s}}$$

8. Use Eq. 31-11 with $v = c$.

$$\frac{E_0}{B_0} = c \rightarrow B_0 = \frac{E_0}{c} = \frac{0.57 \times 10^{-4}\ \text{V/m}}{3.00 \times 10^8\ \text{m/s}} = \boxed{1.9 \times 10^{-13}\ \text{T}}$$

9. Use Eq. 31-11 with $v = c$.

$$\frac{E_0}{B_0} = c \rightarrow E_0 = B_0 c = \left(12.5 \times 10^{-9}\ \text{T}\right)\left(3.00 \times 10^8\ \text{m/s}\right) = \boxed{3.75\ \text{V/m}}$$

10. The frequency of the two fields must be the same: $\boxed{80.0\ \text{kHz}}$. The rms strength of the electric field can be found from Eq. 31-11 with $v = c$.

$$E_{\text{rms}} = cB_{\text{rms}} = \left(3.00 \times 10^8\ \text{m/s}\right)\left(7.75 \times 10^{-9}\ \text{T}\right) = \boxed{2.33\ \text{V/m}}$$

The electric field is perpendicular to both the direction of travel and the magnetic field, so the electric field oscillates along the $\boxed{\text{horizontal north-south line.}}$

12. The wave equation to be considered is $v^2 \frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2}$.

(*a*) Given $E(x,t) = Ae^{-\alpha(x-vt)^2}$.

$$\frac{\partial E}{\partial x} = Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)\right]$$

$$\frac{\partial^2 E}{\partial x^2} = Ae^{-\alpha(x-vt)^2}(-2\alpha) + Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)\right]^2 = -2\alpha Ae^{-\alpha(x-vt)^2}\left[1-2\alpha(x-vt)^2\right]$$

$$\frac{\partial E}{\partial t} = Ae^{-\alpha(x-vt)^2}\left[-2\alpha(x-vt)(-v)\right] = Ae^{-\alpha(x-vt)^2}\left[2\alpha v(x-vt)\right]$$

$$\frac{\partial^2 E}{\partial t^2} = Ae^{-\alpha(x-vt)^2}\left(-2\alpha v^2\right) + Ae^{-\alpha(x-vt)^2}\left[2\alpha v(x-vt)\right]^2 = -2\alpha v^2 Ae^{-\alpha(x-vt)^2}\left[1-2\alpha(x-vt)^2\right]$$

We see that $v^2 \frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2}$, and so the wave equation is satisfied.

(*b*) Given $E(x,t) = Ae^{-\left(\alpha x^2 - vt\right)}$.

$$\frac{\partial E}{\partial x} = Ae^{-\left(\alpha x^2 - vt\right)}(-2\alpha x)$$

$$\frac{\partial^2 E}{\partial x^2} = Ae^{-\left(\alpha x^2 - vt\right)}(-2\alpha) + Ae^{-\left(\alpha x^2 - vt\right)}(-2\alpha x)^2 = -2\alpha Ae^{-\left(\alpha x^2 - vt\right)}\left[1-2\alpha x^2\right]$$

$$\frac{\partial E}{\partial t} = Ave^{-\left(\alpha x^2 - vt\right)} \ ; \ \frac{\partial^2 E}{\partial t^2} = Av^2 e^{-\left(\alpha x^2 - vt\right)}$$

This does NOT satisfy $v^2 \frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2}$, since $-2\alpha v^2 Ae^{-\left(\alpha x^2 - vt\right)}\left[1-2\alpha x^2\right] \neq Av^2 e^{-\left(\alpha x^2 - vt\right)}$ in general.

15. Use the relationship that $d = vt$ to find the time.

$$d = vt \quad \rightarrow \quad t = \frac{d}{v} = \frac{\left(1.50 \times 10^{11}\,\text{m}\right)}{\left(3.00 \times 10^{8}\,\text{m/s}\right)} = 5.00 \times 10^{2}\,\text{s} = \boxed{8.33\,\text{min}}$$

16. Use Eq. 31-14 to find the wavelength.

$$c = \lambda f \quad \rightarrow \quad \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)}{\left(8.56 \times 10^{14}\,\text{Hz}\right)} = \boxed{3.50 \times 10^{-7}\,\text{m}} = 311\,\text{nm}$$

This wavelength is just outside the violet end of the visible region, so it is $\boxed{\text{ultraviolet.}}$

17. (*a*) Use Eq. 31-14 to find the wavelength.

$$c = \lambda f \quad \rightarrow \quad \lambda = \frac{c}{f} = \frac{\left(3.00 \times 10^{8}\,\text{m/s}\right)}{\left(1.00 \times 10^{3}\,\text{Hz}\right)} = \boxed{3.00 \times 10^{5}\,\text{m}}$$

(*b*) Again use Eq. 31-14, with the speed of sound in place of the speed of light.

$$v = \lambda f \quad \rightarrow \quad \lambda = \frac{v}{f} = \frac{\left(341\,\text{m/s}\right)}{\left(1.00 \times 10^{3}\,\text{Hz}\right)} = \boxed{0.341\,\text{m}}$$

(*c*) $\boxed{\text{No}}$, you cannot hear a 1000-Hz EM wave. It takes a pressure wave to excite the auditory system. However, if you applied the 1000-Hz EM wave to a speaker, you could hear the 1000-Hz pressure wave.

20. (*a*) The general form of a plane wave is given in Eq. 31-7. For this wave, $E_x = E_0 \sin\left(kz - \omega t\right)$.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.077\,\text{m}^{-1}} = 81.60\,\text{m} \approx \boxed{82\,\text{m}}$$

$$f = \frac{\omega}{2\pi} = \frac{2.3 \times 10^{7}\,\text{rad/s}}{2\pi} = 3.661 \times 10^{6}\,\text{Hz} \approx \boxed{3.7\,\text{MHz}}$$

Note that $\lambda f = \left(81.60\,\text{m}\right)\left(3.661 \times 10^{6}\,\text{Hz}\right) = 2.987 \times 10^{8}\,\text{m/s} \approx c$.