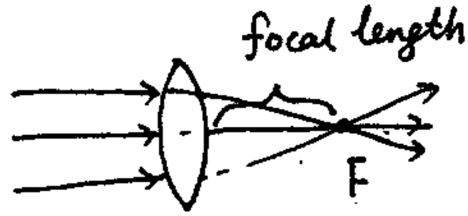
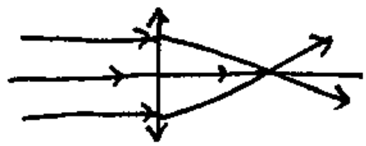
Lect 8 Thin lens

☆ convex lens

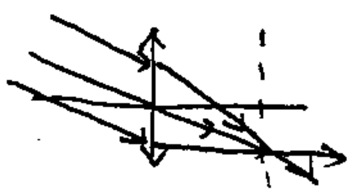


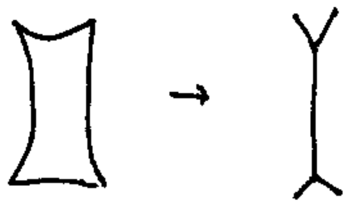
focal point

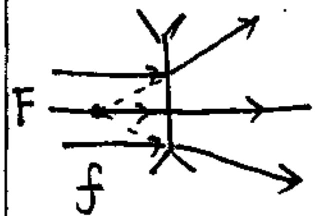


and focal length, focal plane

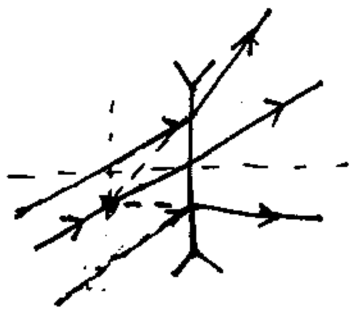
$f > 0$



concave lens 



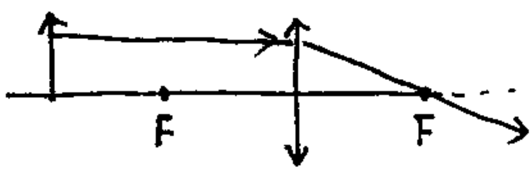
$f < 0$

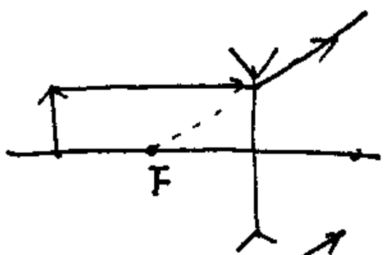


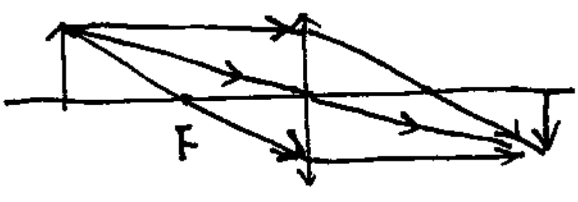
power $p = \frac{1}{f}$ diopter $1D = 1m^{-1}$.

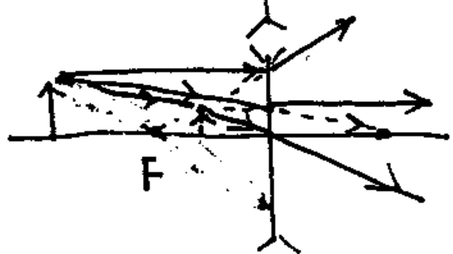
☆ Thin lens equations

3 special ray lines

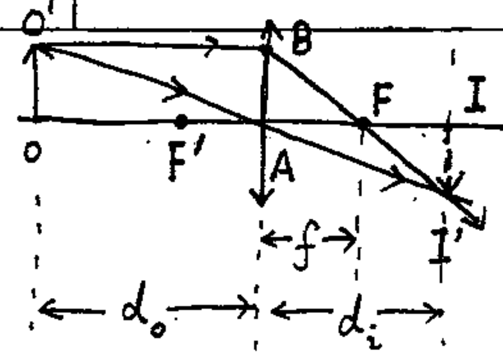
① 



② 



Virtual image at $d_i > f$



- $\Delta F I'I \sim \Delta FBA$

  $\Rightarrow \left|\frac{h_i}{h_o}\right| = \frac{d_i - f}{f}$

  $\Delta O'OA \sim \Delta I'IA \Rightarrow \left|\frac{h_i}{h_o}\right| = \frac{d_i}{d_o}$

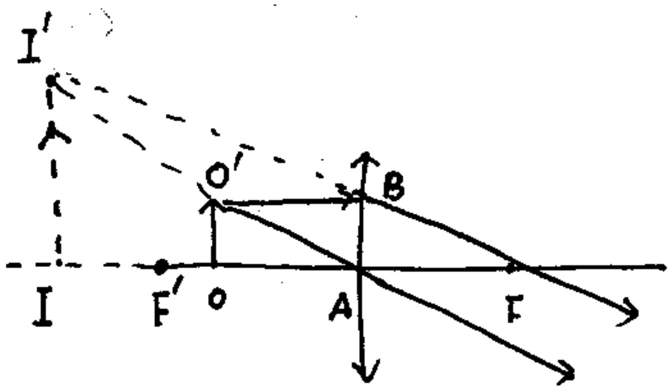
  $\Rightarrow \frac{d_i - f}{f} = \frac{d_i}{d_o} \Rightarrow \frac{d_i}{f} - 1 = \frac{d_i}{d_o} \Rightarrow \boxed{\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}}$

  latteral magnification $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

  notice the negative sign for inverted image.

- imaginary image $d_i < f$

  Similarly $\Delta F I'I \sim \Delta FBA$



  $\left|\frac{h_i}{h_o}\right| = \frac{f - d_i}{f}$ where $d_i < 0$.

  $\Delta O'OA \sim \Delta I'IA$

  $\Rightarrow \left|\frac{h_i}{h_o}\right| = \frac{-d_i}{d_o} \Rightarrow \frac{f - d_i}{f} = -\frac{d_i}{d_o} \Rightarrow \boxed{\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}}$

  $m = -\frac{d_i}{d_o} > 0$ upright

- Summary:

  $d_o > 2f \Rightarrow d_o > d_i > f \Rightarrow m < 0$ and $|m| < 1$

  $2f > d_o > f \Rightarrow d_i > d_o > f \Rightarrow m < 0$ and $|m| > 1$

  $f > d_o > 0 \Rightarrow |d_i| > d_o \Rightarrow m > 1$

Similar formula works for concave lens, but $f<0$ for the case of concave lens.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \Rightarrow \frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}<0$$ $\Rightarrow$ always imaginary image

$$0<m=-\frac{d_i}{d_o}=\frac{-1}{d_o\left(\frac{1}{f}-\frac{1}{d_o}\right)}=\frac{1}{1-\frac{d_o}{f}}=\frac{1}{1+\left|\frac{d_o}{f}\right|}<1.$$

Example: 7.6 cm leaf, 1.00 m from a convex lens with $f=50.\text{mm}$

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{5\text{cm}}-\frac{1}{100\text{cm}}=\frac{19}{100\text{ cm}} \Rightarrow d_i \approx 5.26 \text{ cm}$$

$$m=-\frac{d_i}{d_o}=-\frac{5.26}{100}=-0.0526$$

$$h_i = m\, h_o \approx -7.6\times 0.0526 \approx -0.4 \text{ cm}.$$

Q: if the leaf moves closer to the lens, how does the image move?

Example: object 10 cm before a lens with $f=15$ cm, —

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{15\text{cm}}-\frac{1}{10\text{cm}}=\frac{-1}{30}\text{ cm} \Rightarrow d_i=-30 \text{ cm}$$

$$m=-\frac{d_i}{d_o}=3$$

Ex: a two-lens system

$f_A = 20.0$ cm $f_B = 25.$ cm

the distance between them 80 cm

An object is put 60.0 cm in front of the first lens.

① The first step: $d_o = 60$ cm $f_A = 20.0$ cm

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{20\text{cm}} - \frac{1}{60\text{cm}} = \frac{2}{60\text{cm}} \Rightarrow d_i = 30 \text{ cm}$$

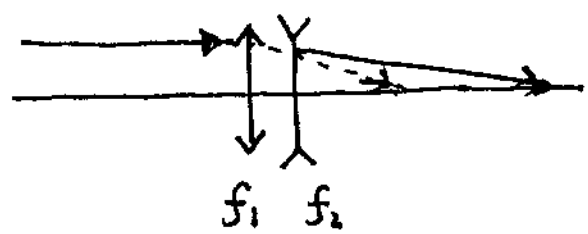
$$m_1 = -\frac{d_i}{d_o} = -\frac{30}{60} = -\frac{1}{2}$$

② second step: $d_o' = 80 - 30 = 50$ cm

$$\frac{1}{d_i'} = \frac{1}{f'} - \frac{1}{d_o'} = \frac{1}{25} - \frac{1}{50} = \frac{1}{50} \Rightarrow d_i' = 50 \text{ cm}$$

$$\Rightarrow m_2 = -\frac{d_i'}{d_o'} = -\frac{50}{50} = -1 \Rightarrow \boxed{m = m_1 m_2 = +1/2.}$$

Ex.

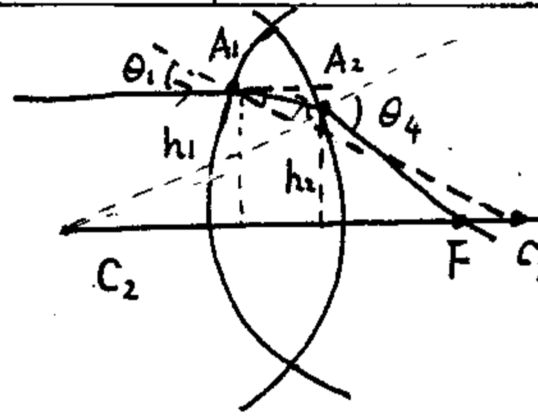


parallel light pass the first convex lens, will focus at the right hand side f_1. This can be considered as a virture object with $d_o = -f_1$ for the next lens

$$\Rightarrow \frac{1}{d_o} + \frac{1}{d_i} = -\frac{1}{|f_2|} \Rightarrow \frac{1}{d_i} = \frac{1}{|f_1|} - \frac{1}{|f_2|} \Rightarrow d_i = \frac{|f_1 f_2|}{|f_2| - |f_1|}$$

$\Rightarrow$ if $f_1 > f_2$, the new focus $f = d_i = -\frac{|f_1||f_2|}{|f_1| - |f_2|} < 0$, otherwise $f = d_i > 0$.

define $\angle A_2A_1C_1 = \theta_2$, $\angle A_1A_2C_2 = \theta_3$



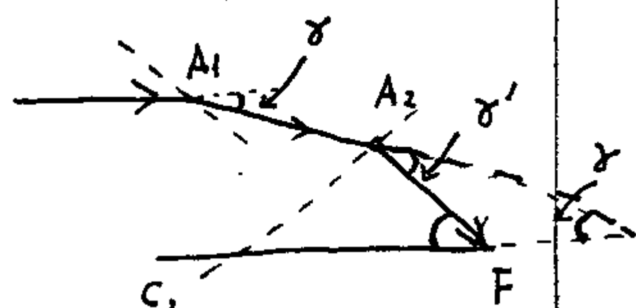
Snell's law

$$\theta_1 = n\theta_2 \ , \quad \theta_4 = n\theta_3$$

$$\theta_1 = \angle A_1C_1C_2 \Rightarrow \theta_1 \approx \sin\theta_1 \approx \frac{h_1}{R_1}$$

$$\angle A_2C_2C_1 \approx \frac{h_2}{R_2}$$

$$\angle A_2FC_2 \approx \frac{h_2}{f}$$



$$\angle A_2FC_2 = \gamma + \gamma'$$

$$\gamma = \theta_1 - \theta_2 = \left(1-\frac{1}{n}\right)\theta_1 \ , \quad \gamma' = \theta_4 - \angle A_1A_2C_2 = \left(1-\frac{1}{n}\right)\theta_4$$

$$\Rightarrow \frac{h_2}{f} = \left(1-\frac{1}{n}\right)\theta_1 + \left(1-\frac{1}{n}\right)\theta_4$$

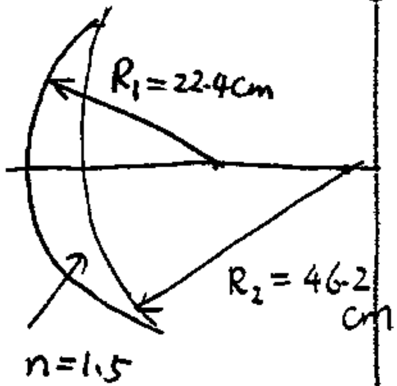
$$\theta_4 = \angle A_2C_2F + \angle A_2FC_2 \Rightarrow \theta_4 = \frac{h_2}{R_2} + \frac{h_2}{f}$$

$$\Rightarrow \frac{h_2}{f} = \left(1-\frac{1}{n}\right)\left[\frac{h_1}{R_1} + \frac{h_2}{R_2} + \frac{h_2}{f}\right]$$

$$\Rightarrow \frac{1}{n}\frac{h_2}{f} = \left(1-\frac{1}{n}\right)\left[\frac{h_1}{R_1} + \frac{h_2}{R_2}\right] \qquad h_1 \simeq h_2 \text{ thin film}$$

$$\Rightarrow \frac{1}{n}\frac{1}{f} = \frac{n-1}{n}\left[\frac{1}{R_1} + \frac{1}{R_2}\right] \Rightarrow \boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right]}$$

Ex: R_2 has opposite curvature, which should be assigned negative sign $R_2 = -46.2\text{ cm}$



$$\Rightarrow \frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= 0.5\left[\frac{1}{22.4} - \frac{1}{46.2}\right] \Rightarrow f = 87.0\text{ cm.}$$