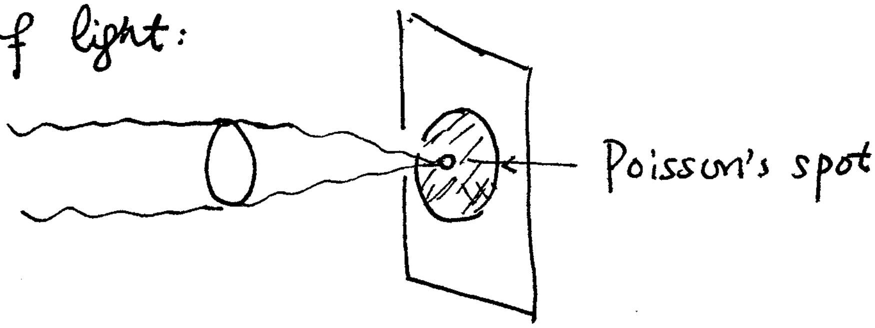
Lect 11 Diffraction (I)

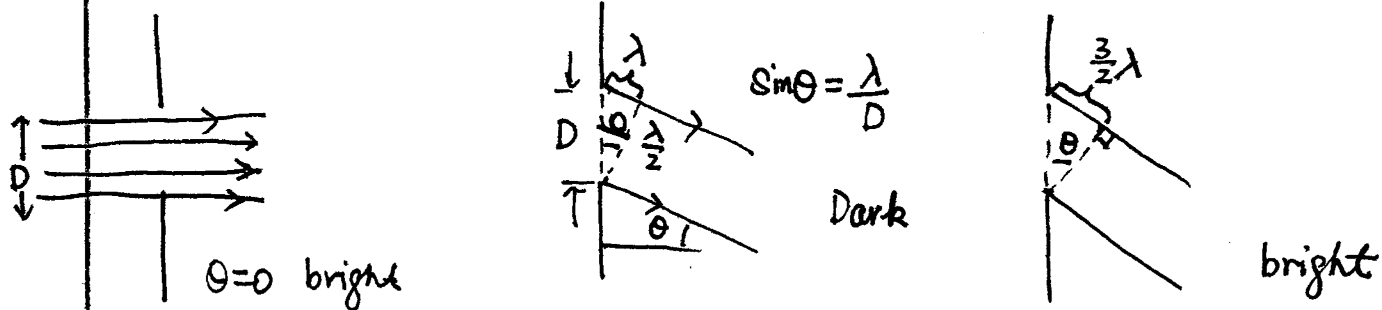
Fresnel: wave theory of light:

Poisson:

Arago



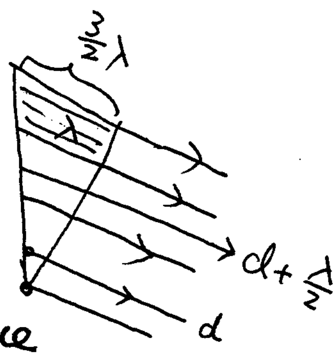
Diffraction by a single slit or Disk



two Rays with path differente $\frac{\lambda}{2}$ cancel each other.

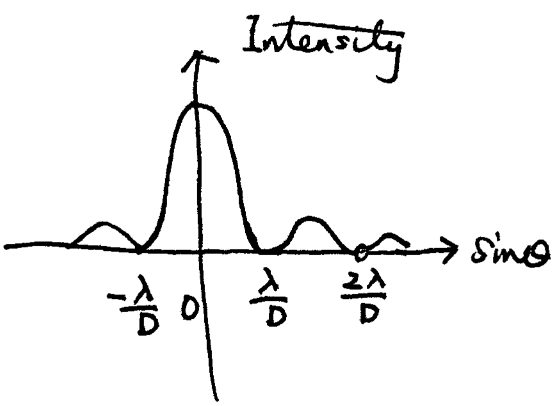
⇒ if $D \sin\theta = n\lambda$ and $n \neq 0$

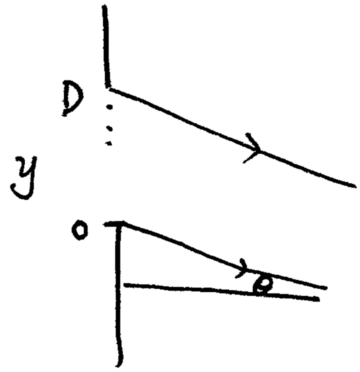
we get perfect cancelation. ⇒ intensity = 0



$D \sin\theta \approx (n + \frac{1}{2})\lambda$, constructive interference

Intensity pattern





$$I \propto \left| \int_0^D dy \; e^{i \frac{y\sin\theta}{\lambda} \cdot 2\pi} \right|^2$$

$$\int_0^D dy \; e^{i\frac{y\sin\theta}{\lambda}\cdot 2\pi} = \frac{\lambda}{2\pi\sin\theta}\left[e^{i\frac{D\sin\theta\cdot 2\pi}{\lambda}} - 1\right]$$

$$= \frac{\lambda}{2\pi\sin\theta}\, e^{i\frac{D\sin\theta\pi}{\lambda}}\; 2i\sin\left(\frac{D\sin\theta\,\pi}{\lambda}\right)$$

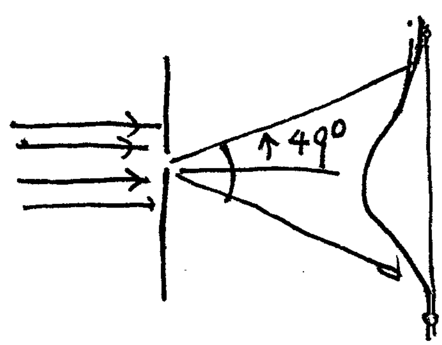
$$\Rightarrow I \propto \left|\frac{\sin\left(\frac{D\sin\theta\pi}{\lambda}\right)}{\frac{\sin\theta\pi}{\lambda}}\right|^2 \Rightarrow \frac{I(\theta)}{I(0)} = \left|\frac{\sin \chi/2}{\frac{\chi}{2}}\right|^2, \quad \text{with } \chi = \frac{D\sin\theta}{\lambda}2\pi$$

The minima occur at $\frac{\chi}{2}=n\pi$, or $\boxed{\frac{D\sin\theta}{n}=\lambda}$.

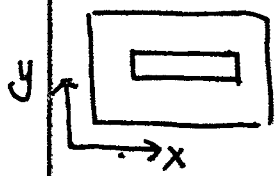
Example: light $\lambda = 750\,nm$, slit $1.0\times10^{-3}\,mm$.

The first minimum at $\sin\theta = \frac{\lambda}{D} = \frac{750\times10^{-9}}{1\times10^{-3}\times10^{-3}} = 750\times10^{-3} = 0.75$

$\Rightarrow \theta = 49^\circ$



② Diffraction spreads

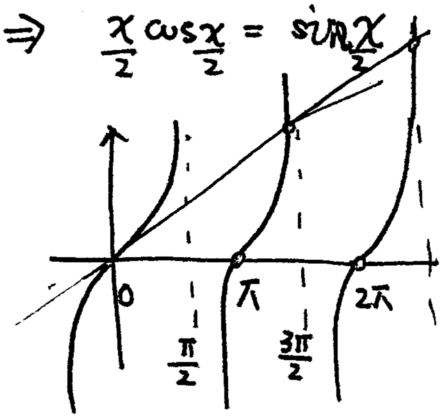


the diffraction pattern spreads more along y than x-direction

③ Intensity at second minima

$$\left[\left(\frac{\sin \chi/2}{\chi/2}\right)^2\right]' = \frac{\sin\chi/2}{\frac{\chi}{2}}\left[\frac{\cos(\chi/2)}{(\chi/2)} - \frac{\sin\chi/2}{\left(\frac{\chi}{2}\right)^2}\right]\frac{1}{2} = 0 \Rightarrow \frac{\chi}{2}\cos\frac{\chi}{2} = \sin\frac{\chi}{2}$$

or $\tan\frac{\chi}{2} = \frac{\chi}{2}$ for maximal



The crossing point is very close to $(m+\frac{1}{2})\pi$.

$$\Rightarrow \frac{I_m}{I_0} = \left|\frac{\sin(m+\frac{1}{2})\pi}{(m+\frac{1}{2})\pi}\right|^2 = \left[\frac{1}{(m+\frac{1}{2})\pi}\right]^2$$

$\frac{I_1}{I_0} = 0.045$, $\frac{I_2}{I_0} = 0.016$. $\Rightarrow$ most intensity goes to the central peak.

* Diffraction in the double-slit experiment.

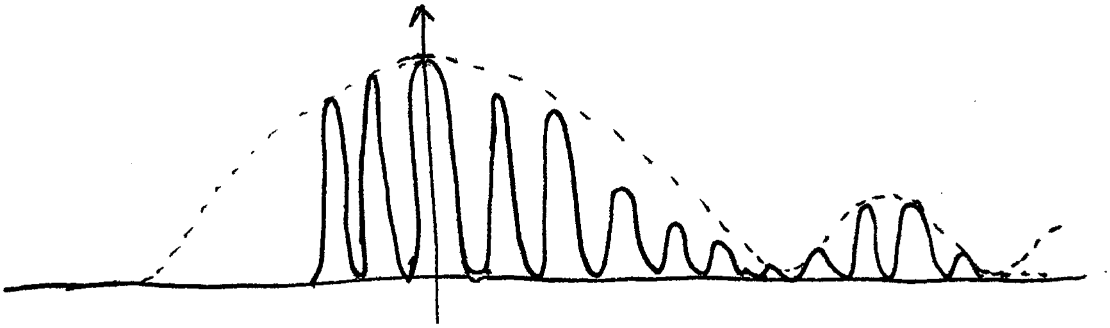
if for each slit, we have E_0 for the total amplitude,

$\Rightarrow$ double slit add together $E_{double} = 2E_0\cos\frac{\delta}{2}$, where $\delta = \frac{d\sin\theta}{\lambda}\cdot 2\pi$

$\Rightarrow$ $E_0 \propto \frac{\sin\chi/2}{\chi/2}$, where $\chi = \frac{D\sin\theta}{\lambda}\cdot 2\pi$ $\Leftarrow$ single slit factor

$$\Rightarrow \frac{I}{I_0} = \left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 \left(\cos\frac{\delta}{2}\right)^2 \qquad \text{where } x = \frac{D\sin\theta}{\lambda}\cdot 2\pi$$

$$\delta = \frac{d\sin\theta}{\lambda}\cdot 2\pi \text{ .}$$



interference + diffraction

$d = 6D = 60\lambda$

$\Rightarrow$ single slit diffraction $\quad \sin\theta = \frac{\lambda}{D} = \frac{.1}{10} \Rightarrow$

double slit $\quad d\sin\theta = \ 6D\sin\theta = 6\lambda$

$\Rightarrow$ there are $2\times 6+1 \ - \underset{\uparrow}{2} \ = 11$

for $m = \pm 6$, dark by the single slit factor.

within the central differaction peak, there are $\longleftarrow$

$m = 0, \pm 1, \cdots \pm 5$, in total 11 bright fringes.

§ limit of resolution

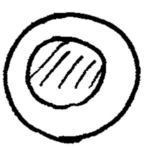
lens : aberrations — a point's image is a tiny blob

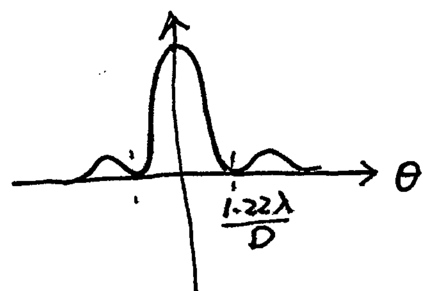
diffraction : — due to the edge of lens

for ~~tens~~ rectangular slit $\Rightarrow$ $\sin\theta = \frac{\lambda}{D}$ (the angular half width of the central peak)

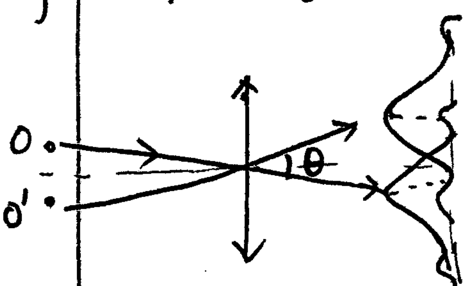
for circular hole $\theta = \frac{1.22\lambda}{D} \sim \frac{\lambda}{D}$

the width is not uniform forom D to 0.





If two point object are very close, we



if one image's center peak is at the other image's minima $\rightarrow$ Rayleigh criterion

angle separation between $O, O' \rightarrow \frac{1.22\lambda}{D}$.

Hubble : $D = 2.4\,m$, $\lambda = 550\,nm$

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22\ 550\times10^{-9}}{2.4} = 2.8\times10^{-7}\ \text{rad}$$

eye resolution : suppose you at airplane 10,000 m, the distance of resolution

$$\theta = \frac{x}{h} = \frac{1.22\lambda}{D}$$

D : pupil 3.0 mm

$\lambda = 550\ nm$

$$\Rightarrow x = h\theta = 1\times10^{4}\times\frac{1.22\times550\times10^{-9}}{3\times10^{-3}} = 2.2\,m.$$

resolution of telescope : ① $D = 200\text{-inch} \simeq 5.1\,m$

$\lambda = 550\,nm \Rightarrow \theta = \frac{1.22\lambda}{D} = 1.3\times10^{-7}$ rad

② radio-telescope $D = 300m$, $\lambda = 4\,cm$

$$\theta = \frac{1.22\times0.04}{300} = 1.6\times10^{-4}\text{ rad}$$

for a micoscope : usually objects are put near the focal plane

$$\text{Resolving power (RP)} = \delta = f\theta = \frac{1.22\lambda f}{D}$$

lens-maker Eq $\frac{1}{f} = (n+1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

$D < R$, $n+1 \sim 2$ $\Big\} \Rightarrow f \sim R > \frac{D}{2}$

$$\Rightarrow S > \frac{1.22}{2}\lambda \approx \frac{\lambda}{2} \Rightarrow \boxed{\text{microscope resolve power} \sim \lambda .}$$

∘ human eyes : $\lambda = 550\,nm$. pupil D $0.1\,cm \sim 0.8\,cm$

$$\Rightarrow \theta \simeq \frac{1.22\lambda}{D} \sim (8\times10^{-5} \sim 6\times10^{-4})\text{ rad}$$

but the structure of eye, fovea : resolution $3\,\mu m$

Length of eye ball $2\,cm$

$$\Rightarrow \theta_{max} \sim \frac{3\times10^{-6}}{2\times10^{-2}} \approx 1\times10^{-4}$$

$\Rightarrow$ aberration ∘ constrains $\theta_{resol} = 5\times10^{-4}$ rad (best eye)

At near point of eye 25cm $\Rightarrow$ $\boxed{S \approx 25cm \times 5\times10^{-4} \approx 0.1\,mm}$

for $\lambda = 400nm$. the micoscope resoultion $\boxed{s \sim \frac{\lambda}{2} = 200nm}$

$\Rightarrow$ the useful magnifcation $\sim \frac{0.1\,mm}{200nm} \approx 500$.

Any greater magnifcation only magnify the diffraction pattern to be visible, and does not increase resolving power.

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