Supplemental Material — correlation functions of Ising model

1. Quantum Ising model at critical point

$$H = -K \sum_i \left[g\,\sigma_x(i) + \sigma_z(i)\,\sigma_z(i+1) \right]$$

with the correspondence to the 2D classic Ising model

$$\boxed{\begin{cases} K\Delta\tau = \beta J \\ \sinh(2\beta J g)\,\sinh(2\beta J) = 1 \end{cases}} \Leftarrow \quad \text{from} \begin{cases} Kg = h\Delta\tau \\ \sinh(2h\Delta\tau)\,\sinh(2\beta J) = 1 \end{cases}$$

at the crictical T_c of 2D Ising model, $\sinh(2\beta_c J) = 1$

$\Rightarrow$ $g_c = 1$ at quantum critical point of 1D quantum Ising model.

2. Define a Jordan-Wigner transformation

$$\begin{cases} \xi_1(n) = \dfrac{1}{\sqrt{2}} \left(\prod_{i<n} \sigma_x(i) \right) \sigma_y(n) \\ \xi_2(n) = \dfrac{1}{\sqrt{2}} \prod_{i<n} \sigma_x(i)\, \sigma_z(n) \end{cases}$$

check $\{ \xi_i(n), \xi_j(m) \} = \delta_{ij}\,\delta_{mn}$

① if $n \neq m$

Proof: Suppose $n < m$, then $\quad a, b = y$ or z

$$\rightarrow \xi_i(n)\, \xi_j(m) = \frac{1}{2}\left[\prod_{l<n} \sigma_x(l)\right] \sigma_a(n) \left[\prod_{l'<m} \sigma_x(l')\right] \sigma_b(m)$$

$$= \frac{1}{2}\left[\prod_{l<n} \sigma_x(l)\right]\left[\prod_{l'<m} \sigma_x(l')\right] (-)\, \sigma_a(n)\, \sigma_b(m)$$

$$= -\frac{1}{2}\left[\prod_{l'<m} \sigma_x(l')\right]\left[\prod_{l<n} \sigma_x(l)\right] \sigma_b(m)\, \sigma_a(n)$$

$$= -\frac{1}{2}\left[\prod_{l'<m} \sigma_x(l')\right] \sigma_b(m) \left[\prod_{l<n} \sigma_x(l)\right] \sigma_a(n) = -\xi_j(m)\, \xi_i(n)$$

② if $n = m$

$$\xi_i(n)\, \xi_j(n) = \frac{1}{2}\left[\prod_{l<n} \sigma_x(l)\right] \sigma_a(n) \left[\prod_{l'<n} \sigma_x(l')\right] \sigma_b(n)$$

$$= \frac{1}{2}\left[\prod_{l<n} \sigma_x(l)\right]^2 \sigma_a(n)\, \sigma_b(n) = \frac{1}{2}\, \sigma_a(n)\, \sigma_b(n)$$

$$\Rightarrow \quad \xi_i(n)\, \xi_j(n) + \xi_j(n)\, \xi_i(n) = \delta_{ij}$$

Combine together $\Rightarrow$ $\boxed{\{\xi_i(n), \xi_j(m)\} = \delta_{ij}\, \delta_{mn}}$

and $\boxed{\psi_1^2(n) = \psi_2^2(n) = 1/2.}$

Majorana fermion operator

(*) Some relations

$$\sigma_x(n) = -i\, \sigma_y(n)\, \sigma_z(n) = -2i\, \xi_1(n)\, \xi_2(n)$$

$$\xi_1(n)\xi_2(n+1) = \frac{1}{2}\left(\prod_{i<n}\sigma_x(i)\right)\sigma_y(n)\left(\prod_{i<n+1}\sigma_x(i')\right)\sigma_z(n+1)$$

$$= \frac{1}{2}\left(\prod_{i<n}\sigma_x(i)\right)^2 \sigma_y(n)\,\sigma_x(n)\,\sigma_z(n+1) = -\frac{i}{2}\sigma_z(n)\,\sigma_z(n+1)$$

Set $K = 1/2$, $\Delta\tau = 2\beta J$

$$\boxed{\begin{aligned} H &= -\frac{1}{2}\sum_i g\,\sigma_x(i) + \sigma_z(i)\,\sigma_z(i+1) \\ &= \sum_i ig\,\xi_1(i)\xi_2(i) - i\,\xi_1(i)\,\xi_2(i+1) \end{aligned}}$$

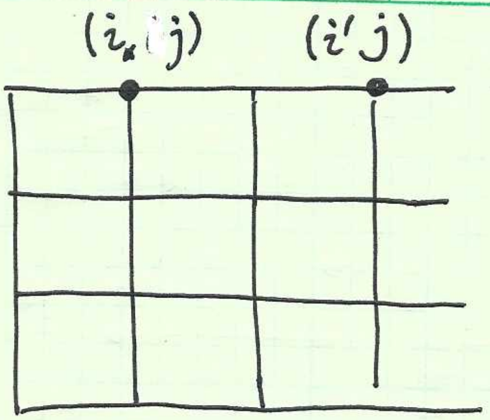
$$H = \frac{1}{2}(1-g)\sum_i \left(-i\,\xi_1(i)\xi_2(i) + i\,\xi_2(i)\xi_1(i)\right)$$

$$+ \frac{1}{2}\sum_i (-i)\,\xi_1(i)\left[\xi_2(i+1) - \xi_2(i)\right] - i\xi_2(i)\left(\xi_1(i) - \xi_1(i-1)\right]$$

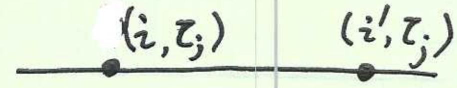
$$\rightarrow \frac{1}{2}\int dx\, \frac{1-g}{a}\,(\xi_1(x), \xi_2(x))\begin{pmatrix} & -i \\ i & \end{pmatrix}\begin{pmatrix}\xi_1 \\ \xi_2\end{pmatrix}(x)$$

$$+ \frac{1}{2}\int dx\, (\xi_1, \xi_2)\begin{pmatrix} & -i\partial_x \\ -i\partial_x & \end{pmatrix}\begin{pmatrix}\xi_1 \\ \xi_2\end{pmatrix}$$

$$H = \frac{1}{2}\int dx\, \xi^T(\alpha P + \beta m)\,\xi, \quad \text{with } \begin{cases} m = \dfrac{1-g}{a} \\ \beta = \sigma_2,\ \alpha = \sigma_1 \end{cases}$$



$\rightarrow$



Classic configuration
$\rightarrow$ world line in the σ_z basis in the QM model

Consider the correlation of σ in the 2D Ising model along the same row

$$\langle \sigma_{(ij)} \sigma_{(i'j)} \rangle = \frac{\sum_{\{\sigma\}} \sigma_{(ij)} \sigma_{(i'j)} e^{\beta J \sum \sigma_{mn} \sigma_{m'n'}}}{\sum_{\{\sigma\}} e^{\beta J \sum \sigma_{mn} \sigma_{m'n'}}}$$

$$= \frac{\sum \mathrm{Tr} [\sigma_z(i,) \sigma_z(i',) T^N]}{\sum \mathrm{Tr} [T^N]} \leftarrow$$

equal time correlation in the QM model

where $T = e^{h \Delta\tau \sum_i \sigma_x(i) + \beta J \sum \sigma_z(i) \sigma_z(i+1)}$

with $\sinh(2h\Delta\tau) \sinh 2\beta J = 1$.

$$\rightarrow G(i,j) = \frac{\sum \langle a | T^N | b \rangle \langle b | \sigma_z(i) \sigma_z(j) | a \rangle}{\sum \langle a | T^N | a \rangle}$$

as $N \to \infty$, we only need to consider the ground ~~state~~ $|0\rangle$ of T, i.e the largest eigenvalue

$$G(ij) \xrightarrow{N\to\infty} \langle 0|\sigma_z(i)\,\sigma_z(j)|0\rangle = \langle 0|\sigma_z(i)\,\sigma_z^2(i+1)\cdots\sigma_z^2(j-1)\,\sigma_z(j)|0\rangle$$

$$= \langle 0|\prod_{l=i}^{j-1}\sigma_z(l)\,\sigma_z(l+1)\,|0\rangle = \langle 0|(2i)\,\xi_1(i)\left[\prod_{l=i+1}^{j-1}(2i)\,\xi_2(l)\,\xi_1(l)\right]\xi_2(j)|0\rangle$$

it can be expressed in terms of Paffian, but it's too complicated

Consider two copies of Ising model "σ" and "S", represented by ξ and η, respectively.

We calculate $\langle\sigma(i)\,S(i)\,\sigma(j)\,S(j)\rangle = \langle\sigma(i)\,\sigma(j)\rangle\langle S(i)\,S(j)\rangle = G^2(i,j)$

$$\Rightarrow G^2(i.j) = \langle 0_s\otimes 0_\sigma|\,(2i)\,\xi_1(i)\left(\prod_{l=i+1}^{j-1}(2i\,\xi_2(l)\,\xi_1(l)\right)\xi_2(j)\otimes(\xi\to\eta)\,|0_s\otimes 0_\sigma\rangle$$

$$= \langle 0_s\otimes 0_\sigma|\,2\eta_1(i)\,\xi_1(i)\left(\prod_{l=i+1}^{j-1}2\,\eta_1(i)\,\xi_1(i)\;2\eta_2(i)\,\xi_2(i)\right)2\eta_2(j)\,\xi_2(j)\;|0_s\otimes 0_\sigma\rangle$$

since $(2\eta\xi)^2 = -1 \Rightarrow e^{\frac{\pi}{2}(2\eta\xi)} = 2\eta\xi$

$$G^2(i,j) = \langle 0_s\otimes 0_\sigma|\,2\eta_1(i)\,\xi_1(i)\,\exp\left[i\pi\sum_{l=i+1}^{j-1}(i)(\xi_1(i)\,\eta_1(i)+\xi_2(i)\,\eta_2(i)\right]$$

$$\cdot\,2\,\eta_2(i)\,\xi_2(i)\,|\,0_s\otimes 0_\sigma\rangle$$

Recall $H_\xi = \int dx \; \frac{-i}{2}[\xi_1 \partial_x \xi_2 + \xi_2 \partial_x \xi_1] - im(\xi_1 \xi_2)$

define chiral basis $\xi_{R,L} = \dfrac{\xi_1 \pm \xi_2}{\sqrt{2}}$

$$\Rightarrow \quad H_\xi = \int dx \; \frac{-i}{2}(\xi_R \partial_x \xi_R - \xi_L \partial_x \xi_L) + im\, \xi_R \xi_L$$

Double the Hamiltonian, define $\eta_{R,L} = \dfrac{\eta_1 \pm \eta_2}{\sqrt{2}}$

$$H_\eta = \int dx \; \frac{-i}{2}(\eta_R \partial_x \eta_R - \eta_L \partial_x \eta_L) + im\, \eta_R \eta_L$$

$$H = H_\xi + H_\eta = \int dx \left\{ \frac{-i}{2}[\xi_R \partial_x \xi_R + \eta_R \partial_x \eta_R - (R \to L)] + im(\xi_R \xi_L + \eta_R \eta_L) \right.$$

Now define $\psi_{R,L} = \dfrac{\xi_{R,L} + i\,\eta_{R,L}}{\sqrt{2}}$, then

$$\psi_R^\dagger \partial_x \psi_R = \frac{1}{2}[\xi_R - i\eta_R]\partial_x[\xi_R + i\eta_R] = \frac{1}{2}\{\xi_R \partial_x \xi_R + \eta_R \partial_x \eta_R] \cdots$$

$$\psi_R^\dagger \psi_L = \frac{1}{2}[\xi_R - i\eta_R][\xi_L + i\eta_L] = \frac{1}{2}[\xi_R \xi_L + \eta_R \eta_L + i(\xi_R \eta_L - \eta_R \xi_L)$$

$$\psi_L^\dagger \psi_R = \frac{1}{2}[\xi_L - i\eta_L][\xi_R + i\eta_R] = \frac{1}{2}[\xi_L \xi_R + \eta_L \eta_R + i[\xi_L \eta_R - \eta_L \xi_R)]$$

$$\Rightarrow \quad \boxed{H = \int dx \;\; \psi_R^\dagger(-i\partial_x)\psi_R + \psi_L^\dagger(i\partial_x)\psi_L + im(\psi_R^\dagger \psi_L - \psi_L^\dagger \psi_R)}$$

T

Then $\xi_{1,2} = \frac{1}{\sqrt{2}}[\xi_R \pm \xi_L]$

$\eta_{1,2} = \frac{1}{\sqrt{2}}[\eta_R \pm \eta_L]$

$$\Rightarrow \xi_1\eta_1 = \frac{1}{2}[\xi_R\eta_R + \xi_L\eta_L + \xi_R\eta_L + \xi_L\eta_R]$$

$$\xi_2\eta_2 = \frac{1}{2}[\xi_R\eta_R + \xi_L\eta_L - \xi_R\eta_L - \xi_L\eta_R]$$

$$\Rightarrow \xi_1\eta_1 + \xi_2\eta_2 = \xi_R\eta_R + \xi_L\eta_L = \frac{1}{2i}(\psi_R + \psi_R^\dagger)(\psi_R - \psi_R^\dagger) + \frac{1}{2i}(\psi_L + \psi_L^\dagger)(\psi_L - \psi_L^\dagger) = \frac{1}{2i}[-\psi_R\psi_R^\dagger + \psi_R^\dagger\psi_R - \psi_L\psi_L^\dagger + \psi_L^\dagger\psi_L]$$

$$= \frac{1}{i}[\psi_R^\dagger\psi_R - 1/2] + \frac{1}{i}[\psi_L^\dagger\psi_L - 1/2] \leftarrow$$

$$\Rightarrow :\psi_R^\dagger\psi_R + \psi_L^\dagger\psi_L: = i\xi_1\eta_1 + i\xi_2\eta_2$$

$$\xi_R\eta_L + \xi_L\eta_R = \frac{1}{2i}[\psi_R + \psi_R^\dagger][\psi_L - \psi_L^\dagger] + \frac{1}{2i}[\psi_L + \psi_L^\dagger][\psi_R - \psi_R^\dagger]$$

$$= \frac{1}{2i}[-\psi_R\psi_L^\dagger + \psi_L^\dagger\psi_R + \psi_R^\dagger\psi_L - \psi_L\psi_R^\dagger] = \frac{1}{i}[\psi_R^\dagger\psi_L + \psi_L^\dagger\psi_R]$$

$$\Rightarrow i\xi_1\eta_1 = \frac{1}{2}:\psi_R^\dagger\psi_R + \psi_L^\dagger\psi_L: + \frac{1}{2}[\psi_R^\dagger\psi_L + \psi_L^\dagger\psi_R]$$

$$i\xi_2\eta_2 = \frac{1}{2}:\psi_R^\dagger\psi_R + \psi_L^\dagger\psi_L: - \frac{1}{2}[\psi_R^\dagger\psi_L + \psi_L^\dagger\psi_R]$$

⊛ Bosonization of the model

$$\psi_R(x) = \frac{1}{\sqrt{2\pi a}} e^{i\sqrt{4\pi}\,\phi_R(x)}, \quad \psi_L = \frac{1}{\sqrt{2\pi a}} e^{-i\sqrt{4\pi}\,\phi_L(x)}$$

$$[\phi_R(x), \phi_R(x')] = \frac{i}{4}\,\mathrm{sgn}(x - x')$$

$$[\phi_L(x), \phi_L(x')] = -\frac{i}{4}\,\mathrm{sgn}(x - x')$$

$$[\phi_R(x)\ \phi_L(x')] = \frac{i}{4}$$

$$\phi(x) = \phi_R(x) + \phi_L(x) \,, \qquad \theta(x) = \phi_R(x) - \phi_L(x)$$

$$[\phi(x), \phi(x')] = [\theta(x), \theta(x')] = 0$$

$$[\phi(x), \theta(x')] = -i\,\Theta(x'-x) = \begin{cases} 0 & x' < x \\ -i & x' > x \end{cases}$$

bosonic variable

$$\rho_R = :\psi_R^\dagger(x+\epsilon)\psi_R(x): \; = \lim_{\epsilon\to 0}\lim_{a\to 0} \left(\psi_R^\dagger(x+\epsilon)\psi_R(x) - \langle|\psi_R^\dagger(x+\epsilon)\psi_R(x)|\rangle\right)$$

$$\psi_R^\dagger(x+\epsilon)\psi_R(x) = \frac{1}{2\pi a} e^{-i\sqrt{4\pi}\,\phi_R(x+\epsilon)} e^{i\sqrt{4\pi}\,\phi_R(x)}$$

$$e^A e^B = :e^{A+B}: \; e^{\langle AB + \frac{A^2+B^2}{2}\rangle}$$

$$= :e^{-i\sqrt{4\pi}\,\epsilon\,\partial_x\phi_R}: \; e^{4\pi\langle \phi_R(x+\epsilon)\phi_R(x) - \phi_R^2(0)\rangle}$$

$$\langle \phi_R(x)\phi_R(x')\rangle = \frac{-1}{4\pi} \ln \frac{2\pi}{L}\left[a - i(x-x')\right] \longleftarrow \text{quote without proof.}$$

$$\Rightarrow e^{4\pi\langle \phi_R(x+\epsilon)\phi_R(x) - \phi_R^2(0)\rangle} = \frac{a}{a-i\epsilon}$$

$$\Rightarrow \rho_R = \lim_{\epsilon\to 0}\lim_{a\to 0} \frac{1}{2\pi a}\left[e^{-i\sqrt{4\pi}\,\epsilon\,\partial_x\phi_R} - 1\right]\frac{a}{a-i\epsilon}$$

$$= \lim_{\epsilon\to 0}\lim_{a\to 0} \frac{1}{2\pi a}\left(-i\sqrt{4\pi}\,\epsilon\,\partial_x\phi_R\right)\frac{a}{a-i\epsilon} = \sqrt{\frac{1}{\pi}}\,\partial_x\phi_R(x)$$

Similarly $\rho_L = \sqrt{\frac{1}{\pi}}\,\partial_x\phi_L(x)$

$$\rho(x) = :\psi_R^\dagger(x)\psi_R(x) + \psi_L^\dagger(x)\psi_L(x): \; = \sqrt{\frac{1}{\pi}}\,\partial_x\phi$$

$$\psi_R^\dagger \psi_L = \frac{1}{2\pi a} e^{-i\sqrt{4\pi}\phi_R} e^{-i\sqrt{4\pi}\phi_L} = \frac{1}{2\pi a} e^{-i\sqrt{4\pi}\phi} e^{-\frac{4\pi}{2}[\phi_R, \phi_L]}$$

$$= \frac{1}{2\pi a} e^{-i\sqrt{4\pi}\phi} e^{-2\pi \frac{i}{4}} = \frac{-i}{2\pi a} e^{-i\sqrt{4\pi}\phi}$$

$$\psi_L^\dagger \psi_R = \frac{i}{2\pi a} e^{i\sqrt{4\pi}\phi}$$

(*) Apply the above result

$$\exp\Big[i\pi \sum_{l=i+1}^{j-1} (\xi_1(l)\eta_1(l) + \xi_2(l)\eta_2(l))\Big] = e^{i\pi \int_i^j dx\, \partial_x \phi}$$

$$= e^{i\sqrt{\pi}\phi(j-a)} e^{-i\sqrt{\pi}\phi(i+a)}$$

$$i\xi_1(i)\eta_1(i) = \frac{a}{2}\Big[\partial_x\phi(i) + \frac{1}{2}\Big[\frac{-i}{2\pi a} e^{-i\sqrt{4\pi}\phi(i)} + \underbrace{\frac{i}{2\pi a} e^{i\sqrt{4\pi}\phi(i)}}\Big]\Big]$$

$$i\xi_2(j)\eta_2(j) = \frac{a}{2}\Big[\partial_x\phi(j) - \frac{1}{2}\Big[\underbrace{\frac{-i}{2\pi a} e^{-i\sqrt{4\pi}\phi(j)}} + \frac{i}{2\pi a} e^{i\sqrt{4\pi}\phi(j)}\Big]\Big]$$

The leading contribution to

$$G^2(i,j) = \langle 0_s \otimes 0_\sigma | \frac{i\,(-i)}{(4\pi)^2} e^{i\sqrt{4\pi}\phi(i)} e^{-i\sqrt{\pi}\phi(i)} e^{i\sqrt{\pi}\phi(j)} e^{-i\sqrt{4\pi}\phi(j)} | 0_s \otimes 0_\sigma \rangle$$

$$\sim \frac{1}{(4\pi)^2} \langle 0_{s\times\sigma} | e^{i\sqrt{\pi}\phi(i)} e^{-i\sqrt{\pi}\phi(j)} | 0_{s\times\sigma} \rangle$$

vertex operator

$$\langle 0| e^{i\beta\phi(x,t)} e^{-i\beta\phi(0)} |0\rangle = \left[\frac{a}{a-i(x-vt)}\right]^{\frac{\beta^2}{4\pi}} \left[\frac{a}{a+i(x+vt)}\right]^{\frac{\beta^2}{4\pi}}$$

$$\xrightarrow{t=0} \left[\frac{a^2}{a^2+x^2}\right]^{\frac{\beta^2}{4\pi}}$$

$$\Rightarrow G^2(i,j) \sim \left(\frac{a^2}{x^2}\right)^{1/4} \sim \frac{1}{x^{1/2}} \leftarrow x = |i-j|$$

$$\Rightarrow G(i,j) \sim \frac{1}{x^{d-2+\eta}} = \frac{1}{x^{1/4}}$$

$$\boxed{\Rightarrow \text{anormalous dimension of 2D Ising model} : \eta = 1/4.}$$

~~How about away from the critical point~~ ?

$$\sinh(2(\beta_c+\Delta\beta)(1+\Delta g)J)\ \sinh(2(\beta_c+\Delta\beta)J) = 1$$

$$\left[\sinh(2\beta_c J) + \cosh(2\beta_c J)[2\Delta\beta J + 2\beta_c \Delta g J]\right]\left[\sinh 2\beta_c + \cosh 2\beta_c\, 2\Delta\beta J\right]$$

$$= 1$$

$$\Rightarrow 2\sinh(2\beta_c J)\cosh 2\beta_c J\ \ 2\Delta\beta J + \cosh 2\beta_c J \cdot (2\beta_c \Delta g J) = 0$$

$$\Rightarrow \Delta g = -\frac{2\sinh(2\beta_c J)\ \Delta\beta}{\beta_c} = +2\sinh(2\beta_c J)\frac{\Delta T}{T_c}$$

The quantum 1D model $m \sim \Delta g \sim \Delta T$

if thes mass term is resposible for the exponetial decay of the magnetic correlation $\xi \sim \frac{1}{m} \sim \frac{1}{\Delta T}$

$\Rightarrow$ $\nu = 1$, rather than the mean field value $\nu = 1/2$.

$\Rightarrow$ 2D Ising model $\boxed{\eta = 1/4, \quad \nu = 1}$

From scaling ~~rela~~ law $\Rightarrow$
$$\begin{cases} \alpha = 0 \\ \beta = 1/8 \\ \gamma = 7/4 \\ \delta = 15 \end{cases}$$

Kink - operator - Majorona operator

$$\sigma_x(n) = -i\,\sigma_y(n)\,\sigma_z(n) = -2i\,\xi_1(n)\,\xi_2(n)$$

$$\sigma_y(n) = \sqrt{2}\,\xi_1(n) \prod_{i<n} \sigma_x(i) = \sqrt{2}\,\xi_1(n) \prod_{i<n} (-2i)\,\xi_1(i)\,\xi_2(i)$$

$$\sigma_z(n) = \sqrt{2}\,\xi_2(n) \prod_{i<n} (-2i)\,\xi_1(i)\,\xi_2(i)$$

$$\mu^z_{n+1/2} = \prod_{j<n} \sigma^x_j = \prod_{j\le n} \left[-2i\,\xi_1(j)\,\xi_2(j)\right]$$

$$\mu^x_{n+1/2} = \sigma^z_n\,\sigma^z_{n+1} = 2i\,\xi_1(n)\,\xi_2(n+1)$$

$$\mu^y_{n+1/2} = -i\,\mu^z_{n+1/2}\,\mu^x_{n+1/2} = \left[\prod_{j\le n-1} \left(-2i\,\xi_1(j)\,\xi_2(j)\right)\right] \overset{(-i)}{(-2i)}\,\xi_1(n)\,\xi_2(n)\,\xi_1(n)\,\xi_2(n+1)$$

$$= \prod_{j\le n-1} \left[-2i\,\xi_1(j)\,\xi_2(j)\right]\; 2\,\xi_2(n)\,\xi_2(n+1)$$

(*) $$\sigma^z_n\,\mu^z_{n-1/2} = \sqrt{2}\,\xi_2(n) \prod_{i<n} (-2i)\,\xi_1(i)\,\xi_2(i) \prod_{j<n} \left[-2i\,\xi_1(j)\,\xi_2(j)\right] = \sqrt{2}\,\xi_2(n)$$

or $$\xi_2(n) = \frac{1}{\sqrt{2}}\,\sigma^z_n\,\mu^z_{n-1/2}$$

Similarly $$\xi_1(n) = \frac{1}{\sqrt{2}\,i}\,\sigma^z_n\,\mu^z_{n+1/2}$$

$$\sigma^z_n\,\xi_2(n) = \frac{1}{\sqrt{2}}\,\mu^z_{n-1/2}, \qquad \sigma^z_n\,\xi_1(n) = \frac{1}{\sqrt{2}\,i}\,\mu^z_{n+1/2}$$

$$\mu^z_{n-1/2}\,\xi_2(n) = \frac{1}{\sqrt{2}}\,\sigma^z_n, \qquad \mu^z_{n+1/2}\,\xi_1(n) = \frac{i}{\sqrt{2}}\,\sigma^z_n$$