Phase-sensitive measurement — d-wave symmetry

The therodynamic anomalies of the d-wave superconductors only detect the linear density of states of nodal quasi-particles. They are not phase sensitive — we need smoking gun evidence for sign-change of gap functions. Below we will see this from Josephson tunneling junction.

According to linear-response theory, the tunneling currents between SCs (see Dan's notes)

$$A = \sum_{kq\sigma} T_{kq}\, C^{\dagger}_{L,k\sigma}\, C_{Rq\sigma}, \longrightarrow A(t) = \sum_{kq\sigma} T_{kq}\, C^{\dagger}_{L,k\sigma}(t) C_{Rq\sigma}(t).$$

$$K_0 = H_0 - \mu_L N_L - \mu_R N_R, \quad \text{and} \quad e\mathcal{U} = \mu_L - \mu_R$$

$$\rightarrow C_{\alpha,k\sigma}(t) = e^{-iK_0 t}\, C_{\alpha,k\sigma}\, e^{iK_0 t}, \quad \alpha = R, L.$$

The tunneling currents $I(t) = I_Q(t) + I_J(t)$

$$I_Q(t) = -\frac{2e}{\hbar^2}\, \mathrm{Im}\int_{-\infty}^{+\infty} dt'\, e^{ie\mathcal{U}(t-t')}\, X_{ret}(t-t') \leftarrow \text{normal current}$$

$$I_J(t) = \frac{2e}{\hbar^2}\, \mathrm{Im}\int_{-\infty}^{+\infty} dt'\, e^{-ie\mathcal{U}(t+t')}\, Y_{ret}(t-t') \leftarrow \text{Josephson tunneling}$$

retarded Green's function

$$X_{ret}(t-t') = -i\theta(t-t')\langle [A(t), A^{\dagger}(t')]\rangle_0$$

$$Y_{ret}(t-t') = -i\theta(t-t')\langle [A^{\dagger}(t)\; A^{\dagger}(t')]\rangle_0 \leftarrow \text{tunneling } L \rightarrow R.$$

⇒ The Josephson channel

$$I_J(t) = \frac{2e}{\hbar^2}\, \mathrm{Im}\left[e^{-2ie\mathcal{U}t}\, Y_{ret}(\Omega = e\mathcal{U})\right]$$

where $y_{ret}(\Omega) = \int_{-\infty}^{+\infty} dt \; e^{i\Omega t} \, y_{ret}(t)$

$$y_{ret}(\Omega) = e^{i(\phi_R+\phi_L)} \sum_{kq} T_{kq} T_{-k-q} \frac{\Delta_L(k)}{E_L(k)} \cdot \frac{\overset{*}{\Delta_R}(q)}{E_R(q)}$$

$$\left\{ \frac{1}{\hbar\Omega + E_L(k) + E_R(q) + i\eta} - \frac{1}{\hbar\Omega - E_L(k) - E_R(q) + i\eta} \right].$$

* Check dimesin $[I] = \frac{e}{\hbar^2} \cdot [\text{time}] \cdot [\text{energy}]^2 = \frac{e}{\hbar} \, e \, [\text{Volt}] = \frac{e^2}{\hbar} \cdot [\text{Voltage}]$

correct : $\frac{e^2}{\hbar}$ is the unit of condutance.

Aussuming T_{kq}'s are momentum indepedent $\Rightarrow$

$$y_{ret}(\Omega) = \frac{\hbar^2 G_N}{2\pi e^2} e^{i(\phi_R - \phi_L)} \int_0^{+\infty} d\xi_L \int_0^{+\infty} d\xi_R \int \frac{d\varphi_L}{2\pi} \int \frac{d\varphi_R}{2\pi}$$

$$\left\{ \frac{\Delta_L(\xi_L, \varphi_L)}{E_L} \frac{\overset{*}{\Delta_R}(\xi_R, \varphi_R)}{E_R} \right\} \left[\frac{1}{\hbar\Omega + E_L + E_R + i\eta} - \frac{1}{\hbar\Omega - E_L - E_R + i\eta} \right]$$

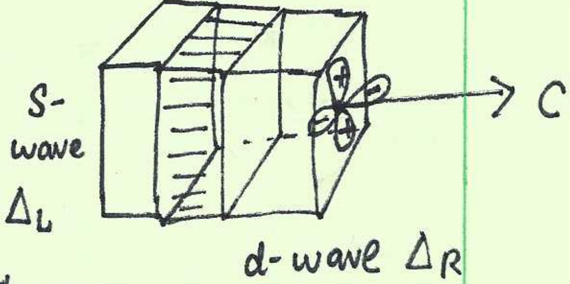
The new property of unconventional SC is the appeareace of angular dependence of

$$\boxed{\int \frac{d\varphi_L}{2\pi} \int \frac{d\varphi_R}{2\pi} \quad \Delta_L(\xi_L, \varphi_R) \, \overset{*}{\Delta_R}(\xi_R, \varphi_R)}$$

(if in 3d system $\int \frac{d\varphi_{L,R}}{2\pi} \rightarrow \int \frac{d\Omega_{L,R}}{4\pi}$)

we can neglect angular dependence in E_L and E_R, because their dependence is $|\Delta_L|^2$ and $|\Delta_R|^2$.

① Tunneling between s-wave and d-wave SC along the c-axis



$$\int \frac{d\varphi_R}{2\pi}\, \Delta_R^*(\xi_R, \varphi_R) = 0 \,!$$

No Josephson tunneling. This result is exact up to second order perturbation theory.

Q: Effective Ginzburg-Landau equation: Can we write down a coupling at the quadratic order?

$\boxed{\text{YBCO is a different story: it's s+d}}$

$\Delta F = -J(\Delta_s^* \Delta_d + c.c.)$ No! this term is not invariant under rotation 90° around c-axis.

but s and d-wave part do can couple at quartic order as

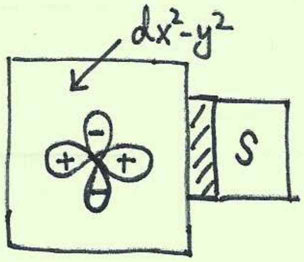
$$\Delta F = -J(\Delta_d^* \Delta_s)^2 + c.c.$$

$$\rightarrow \quad I \propto \sin\left[2(\phi_R - \phi_L + eUt)\right]$$

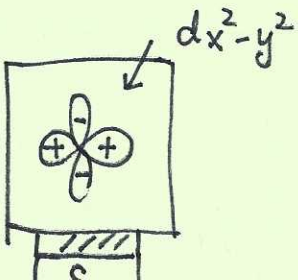
high order Josephson effect, two-pair tunneling.

② however, if the junction is set in the ab-plane due to geometry, we cannot neglect the momentum dependence of $T_{k,q}$.

For example, the tunneling matrix elements for $k \parallel$ surface and $k \perp$ surface are different.



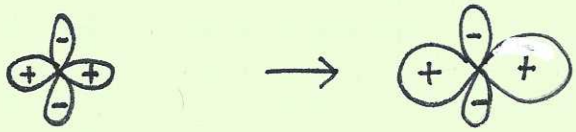
A)



B)

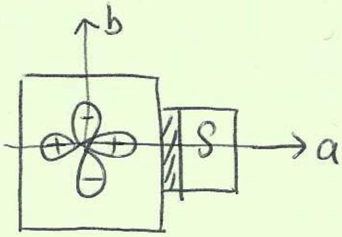
For example, in A) we expect that the tunneling for $\vec{k} \parallel \hat{x}$ is the much stronger than $\vec{k} \parallel \hat{y}$. Thus the d-wave gap function is not averaged evenly, i.e. $\int \frac{d\varphi_L}{2\pi} \, \Delta_L(\xi_L, \varphi_L) \, t(\varphi_L) \neq 0$.

effect from tunneling matrix element.



Δ_L $\qquad\qquad$ $\Delta_L t(\varphi_L)$

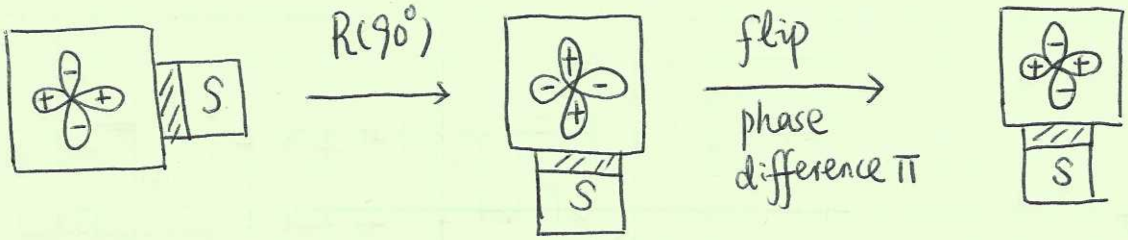
We do have coupling at quadratic level as



$$\Delta F_x = -J \, (\Delta_{s_1}^* \Delta_d + c.c).$$

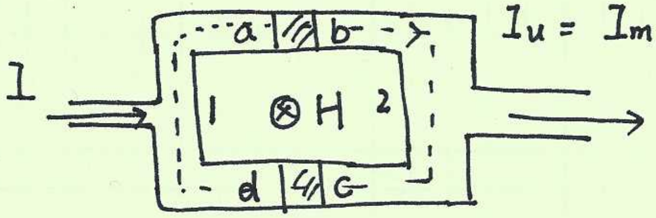
(There's no symmetry about this set-up which can change the sign of the d-wave part!)

- Nevertheless, we can build up the relation between the coupling configurations of A) and B).



$$\Delta F_y = -J \, (\Delta_{s_2}^* \Delta_d + c.c)$$

Digression ①: coupled two-junction SQUID



$$I_u = I_{mu} \sin \tilde{\Delta}\phi_u$$

$$I_d = I_{md} \sin \tilde{\Delta}\phi_d$$

$$I = I_u + I_d = I_{mu} \sin\tilde{\Delta}\phi_u + I_{ml} \sin\tilde{\Delta}\phi_d$$

$\boxed{\text{should be gauge invariant phase difference}}$

If $I_{mu} = I_{ml} = I_m$, we say that these two junctions are matched.

$$\Rightarrow \quad I = 2 I_m \sin \frac{\tilde{\Delta}\phi_u + \tilde{\Delta}\phi_d}{2} \cos \frac{\tilde{\Delta}\phi_u - \tilde{\Delta}\phi_d}{2}$$

$$\oint \nabla\phi \cdot dl = (\phi_b - \phi_a) + (\phi_c - \phi_b) + (\phi_d - \phi_c) + (\phi_a - \phi_b) = 2n\pi$$

The phase difference across the up and down junction

gauge invariant phase: $\boxed{\tilde{\Delta}\phi_u = \phi_b - \phi_a - \left(\int_a^b \vec{A} \cdot dl\right) 2\pi / \Phi_0}$

that enters the formula of current.

$$\Rightarrow \quad \phi_b - \phi_a = \tilde{\Delta}\phi_u + \left(\int_a^b \vec{A} \cdot d\vec{l}\right) \cdot \frac{2\pi}{\Phi_0}$$

$$\phi_d - \phi_c = -\tilde{\Delta}\phi_d + \left(\int_c^d \vec{A} \cdot d\vec{l}\right) \frac{2\pi}{\Phi_0}$$

(phase arcoss the junction).

$$\Phi_0 = \frac{hc}{2e} = 2.07 \times 10^{-7} \text{ Gauss} \cdot \text{cm}^2$$

$$\phi_c - \phi_b = \int_b^c \nabla\phi \cdot dl = \frac{2\pi}{\Phi_0} \int_b^c \left(\vec{A} + \frac{4\pi}{c}\lambda_L^2 \vec{j}\right) \cdot d\vec{l}$$

inside superconductor

$$\phi_a - \phi_d = \int_d^a \nabla\phi \cdot dl = \frac{2\pi}{\Phi_0} \int_d^a \left(\vec{A} + \frac{4\pi}{c}\lambda_L^2 \vec{j}\right) \cdot d\vec{l}$$

Add ~~together~~ $\Rightarrow$ $\Delta\phi_u - \Delta\phi_d + \oint \vec{A}\cdot d\ell \left(\frac{2\pi}{\Phi_0}\right) + \frac{2\pi}{\Phi_0}\frac{4\pi\lambda_L^2}{c}\int_{c'} \vec{j}\cdot d\ell$

$= 2n\pi$

$$\Rightarrow \Delta\phi_u - \Delta\phi_d = -\frac{2\pi}{\Phi_0}\oint \vec{A}\cdot d\vec{\ell} - \frac{2\pi}{\Phi_0}\frac{4\pi\lambda_L^2}{c}\int_{c'} \vec{j}\cdot d\vec{\ell}$$

exclude the insulater junction

we can choose the loop deep inside the superconductor, such that $j = 0$ $\Rightarrow$

$$\Delta\phi_u - \Delta\phi_d = -\frac{2\pi}{\Phi_0}\oint \vec{A}\cdot d\vec{\ell} = -2\pi\,\Phi/\Phi_0$$

$$\Rightarrow I = 2I_m \sin(\Delta\phi_u + \pi\Phi/\Phi_0)\cos\left(\frac{\pi\Phi}{\Phi_0}\right) \qquad ①$$

If the inductance of the loop ~~is~~ is considered, the flux Φ consists two part: $\Phi = \Phi_{ex} + L I_{cir}$ ②

and the circulate current $I_{cir} = I_m(\sin\Delta\phi_u - \sin\Delta\phi_d)$ ③

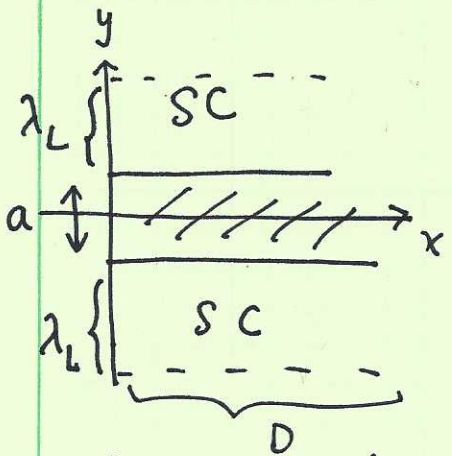
In principle ①, ②, ③ should be solved consistently. If the self-inductance can be neglected, we have

$$I = 2I_m \sin\left(\Delta\phi_u + \pi\frac{\Phi_{ex}}{\Phi_0}\right)\cos\left(\frac{\pi\Phi_{ex}}{\Phi_0}\right),$$

$$\Rightarrow I_{max} = 2I_m\left|\cos\frac{\pi\Phi_{ex}}{\Phi_0}\right|,$$

the maximum supercurrent desity oscillatate with Φ_{ex}/Φ_0.

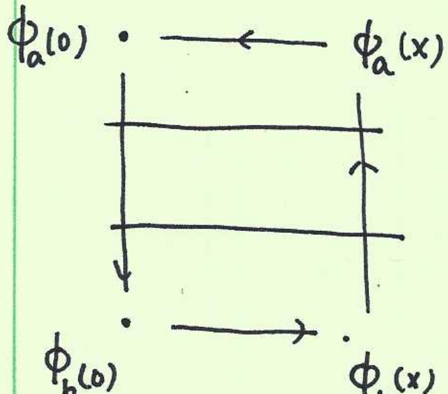
Digression ②: Fraunhofer pattern: a single planar Josephson junction.



$$A_x = 0,$$

$$A_y = \begin{cases} -Bx\, e^{-(y-a/2)/\lambda_L} & y > a/2 \\ -Bx & a/2 > y > -a/2 \\ -Bx\, e^{(y+a/2)/\lambda_L} & y < -a/2 \end{cases}$$

the gauge-independent phase difference



$$\Rightarrow \tilde{\Delta}\phi(x) = \tilde{\Delta}\phi(0) - \frac{2e}{\hbar c}\int_{-\infty}^{+\infty} A_y(x)\, dy$$

$$= \tilde{\Delta}\phi(0) - \frac{2e}{\hbar c} B(a+2\lambda_L)\, x$$

$$\left\{ \begin{aligned} \oint \nabla\phi\, dl = {} & (\phi_a(x) - \phi_b(x)) + (\phi_a(0) - \phi_a(x)) \\ & + (\phi_b(0) - \phi_a(0)) + (\phi_b(x) - \phi_b(0)) = 2n\pi \end{aligned} \right.$$

$$\left. \begin{aligned} \tilde{\Delta}\phi_a(x) &= \phi_a(x) - \phi_b(x) - \frac{2\pi}{\Phi_0}\int_{bx}^{ax} \vec{A}\cdot d\vec{l} \\ \tilde{\Delta}\phi(0) &= \phi_a(0) - \phi_b(0) - \frac{2\pi}{\Phi_0}\int_{b0}^{a0} \vec{A}\cdot d\vec{l} \\ \Rightarrow \quad \phi_a(0) - \phi_a(x) &= -\frac{2\pi}{\Phi_0}\int_{ax}^{a0} \vec{A}\cdot d\vec{l} \quad \cdots \\ \phi_b(x) - \phi_b(0) &= -\frac{2\pi}{\Phi_0}\int_{b0}^{bx} \vec{A}\cdot d\vec{l} \end{aligned} \right\}$$

$$\Rightarrow I = \int_0^D j_y(x)\,dx = j_m \int_0^D dx \; \sin(\tilde{\Delta}\phi(x))$$

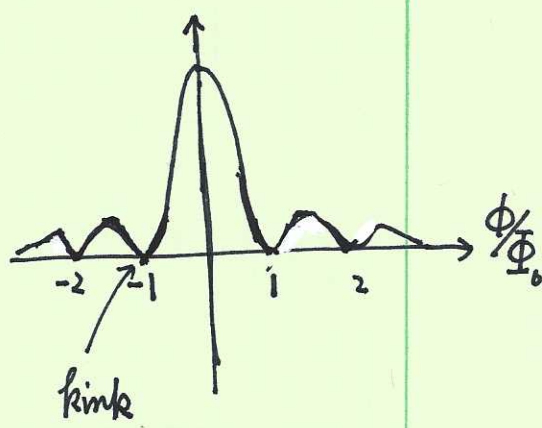
$$= j_m D \int_0^1 d\,x/_D \; \sin\left(\tilde{\Delta}\phi(0) - \frac{2e}{\hbar c} BD(a+2\lambda_L)\, x/_D\right)$$ define $\Phi = BD(a+2\lambda_L)$

$$= j_m D \int_0^1 dx' \; \sin\left(\tilde{\Delta}\phi(0) - 2\pi \frac{\Phi}{\Phi_0} x'\right)$$

$$= j_m D \frac{1}{2\pi\, \Phi/\Phi_0} \left[\cos\left(\tilde{\Delta}\phi(0) - \frac{2\pi}{\Phi_0}\Phi\right) - \cos(\tilde{\Delta}\phi(0))\right]$$

$$= j_m D \frac{\sin \pi\, \Phi/\Phi_0}{\pi \Phi/\Phi_0} \cdot \sin\left(\tilde{\Delta}\phi(0) - \frac{\pi\Phi}{\Phi_0}\right)$$

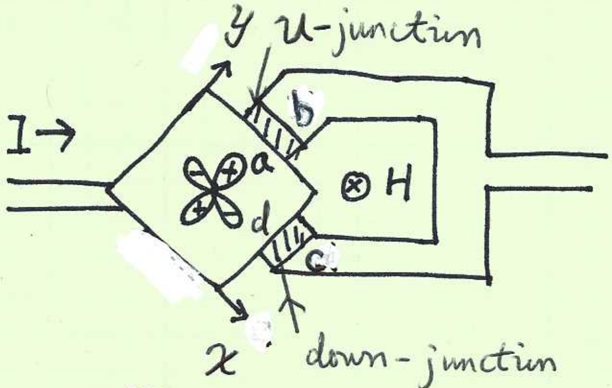
$$\Rightarrow I_{max} = j_m D \left| \frac{\sin \pi\, \Phi/\Phi_0}{\pi\, \Phi/\Phi_0} \right|$$



Now let us apply it to the d-wave case

1. d-S SQUID:

The two junctions have opposit sign of coupling constants



$$I = I_u + I_d = I_m\left(\sin\tilde{\Delta}\phi_u - \sin\tilde{\Delta}\phi_d\right)$$

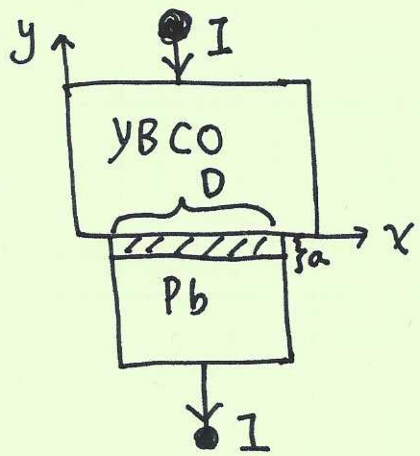
$$= 2I_m \sin\frac{\tilde{\Delta}\phi_u - \tilde{\Delta}\phi_d}{2}\cos\frac{\tilde{\Delta}\phi_u + \tilde{\Delta}\phi_d}{2} = 2I_m \sin\frac{\pi\Phi}{\Phi_0}\cos\left(\Delta\phi_u + \frac{\pi\Phi}{\Phi_0}\right)$$

$$\Rightarrow \boxed{I_{max} = 2I_m\left|\sin\pi\Phi/\Phi_0\right|}$$

The d-S SQUID has maximum current density at $\Phi = \frac{1}{2}\Phi_0$ due to geometric flux π.

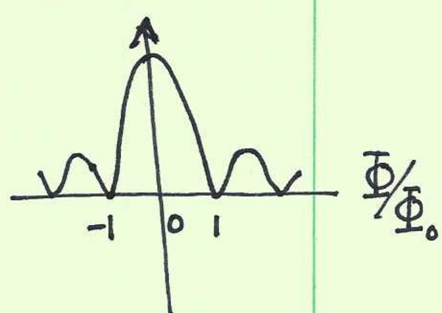
2. Corner junction

if we make a planar Josephsun junction

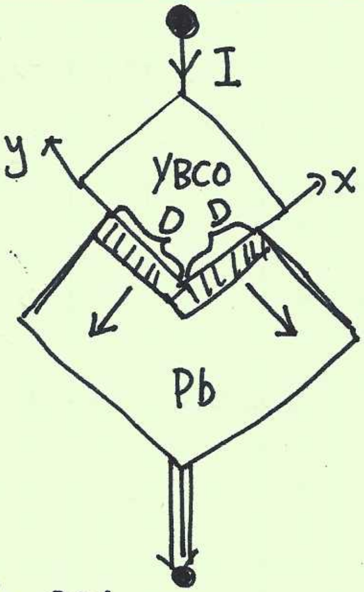


This planar junction will exhibit the same Fraunhofer pattern as two s-wave one, i.e

$$I_{max} \propto \left|\frac{\sin\pi\Phi/\Phi_0}{\pi\Phi/\Phi_0}\right|$$



define $L = a + \lambda_{YBCO} + \lambda_{Pb}$



a: is the width of insulating region

L: is effective width of junction

for x-direction junction, its current projection to 45°-direction is

$$\frac{1}{\sqrt{2}} j_m \int_0^1 dx' \sin\left(\tilde{\Delta}\phi(0) - \frac{2\pi\Phi}{\Phi_0} x'\right) \qquad \Phi = BDL$$

when calculate the current to y-direction, we need to change gauge

$$A_y = 0\,,\quad A_x = By \qquad \left(\text{c.f. the old gauge } \begin{matrix} A_x = 0 \\ A_y = -Bx \end{matrix}\right).$$

$\Rightarrow$ current along 45°: $-\frac{1}{\sqrt{2}} j_m \int_0^1 dy' \sin\left(\tilde{\Delta}\phi(0) + \frac{2\pi\Phi}{\Phi_0} y'\right)$

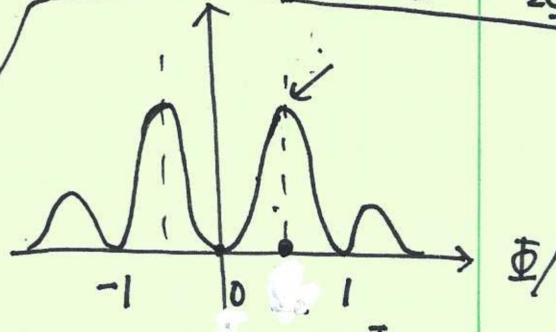
the overall "$-$" sign comes from the d-wave symmetry

$$\Rightarrow I_{tot} = \frac{j_m}{\sqrt{2}} D \frac{\sin \pi\Phi/\Phi_0}{\pi\Phi/\Phi_0} \left[\sin\left(\tilde{\Delta}\phi(0) - \frac{\pi\Phi}{\Phi_0}\right) - \sin\left(\tilde{\Delta}\phi(0) + \frac{\pi\Phi}{\Phi_0}\right)\right]$$

$$= -\sqrt{2} j_m D \frac{\sin^2 \pi\Phi/\Phi_0}{\pi\Phi/\Phi_0} \cos\left(\tilde{\Delta}\phi(0)\right)$$

$$\pi\Phi_a = \tan\left(\frac{\pi\Phi_a}{2\Phi_0}\right)$$

$$\Rightarrow \boxed{I_{max} = I_0 \frac{\sin^2 \pi\Phi/\Phi_0}{\pi\Phi/\Phi_0}}$$



$\Phi_a = 0.742\Phi_0$