3 a) Prove that the 3D plasmon dispersion relation is

$$\omega_q^2 = \omega_p^2 + \frac{3}{5} q^2 v_F^2 \quad \text{where } v_F \text{ is the Fermi velocity}$$

$$\text{and } \omega_p^2 = \frac{4\pi N e^2}{m}$$

(from the evaluation of ~~the~~ zero of $\mathcal{E}(q,\omega)$, at $\frac{\omega}{v_F q} \gg 1$).

b) Derive the plasma frequency through classic equation of motion.

Hint: use the continuity equation and equation of motion

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\vec{v}) = 0, \qquad m\frac{\partial}{\partial t}(n\vec{v}) + m\vec{v}\cdot\nabla(n\vec{v}) = -ne\vec{E}.$$

Expand to the leading order fluctuation $\delta n = n - n_0$. Show

$$\frac{\partial^2}{\partial t^2}\delta n = -\frac{4\pi n_0}{m}\delta n \quad \text{at the leading order.}$$

4) We consider a Jellium model.

a) Show that if the screened interaction is taken as the Thomas-Fermi form, then in the limit of low density region, the Hartree Fock energy of an electron with spin up or down is independent of ~~k~~ its momentum.

b) Assume a finite spin polarization $P = N_\uparrow - N_\downarrow$, find the total Hartree-Fock energy in the limit of a). Express the result as a function of density n, Fermi energy $\mathcal{E}_F^0$, polarization P, and Thomas-Fermi vector k_{TF}.

c) Calculate the spin-susceptibility due to Hartree-Fock correction. Compare it with the free system. Is the system possible to be spin-polarized? (ferro-mag)