HW1

Problem 2.1.6.
Show that the propagator of a harmonic oscillator has the form

$$G(x_b, t, x_a, 0) = A(t) \exp\left(\frac{\mathrm{i} m\omega_0}{2\sin(t\omega_0)}[(x_b^2 + x_a^2)\cos(\omega_0 t) - 2x_b x_a]\right)$$

Use the normalization condition or the path integral to show that

$$A(t) = \left(\frac{m\omega_0}{2\pi \mathrm{i} \sin(t\omega_0)}\right)^{1/2}$$

Problem 2.3.6.
Spin waves: Consider a spin-S quantum spin chain $H = \sum_i J\boldsymbol{S}_i \boldsymbol{S}_{i+1}$. For $J < 0$, the classical ground state is a ferromagnetic state with $\boldsymbol{S}_i = S\hat{z}$. For $J > 0$, it is an anti-ferromagnetic state with $\boldsymbol{S}_i = S\hat{z}(-)^i$.

1. Write down the action for the spin chain.
2. Find the equation of motion for small fluctuations $\delta\boldsymbol{S}_i = \boldsymbol{S}_i - S\hat{z}$ around the ferromagnetic ground state. Transform the equation of motion to frequency and momentum space and find the dispersion relation. Show that $\omega \propto k^2$ for small k.
3. Find the equation of motion for small fluctuations $\delta\boldsymbol{S}_i = \boldsymbol{S}_i - S\hat{z}(-)^i$ around the anti-ferromagnetic ground state. Find the dispersion relation. Show that $\omega \propto |k - \pi|$ for k near to π.