PHYS 212A: Homework 4

November 25, 2013

Exercise 1

a

$$|S|^2 = \frac{1}{1 + \frac{1}{ka}^2} \tag{1}$$

$$|R|^2 = \frac{\frac{1}{ka}^2}{1 + \frac{1}{ka}^2} \tag{2}$$

$$|S|^2 + |R|^2 = \frac{1 + \frac{1}{ka}^2}{1 + \frac{1}{ka}^2} = 1 \tag{3}$$

b

$$j_x|_{0+} - j_x|_{0-} = \frac{\hbar}{2m}(\psi^*(0_+)\psi'(0_+) - \psi(0_+)\psi'^*(0_+)) - \frac{\hbar}{2m}(\psi^*(0_-)\psi'(0_-) - \psi(0_-)\psi'^*(0_-)) \tag{4}$$

Now $\psi(0_-) = \psi(0_+) = \psi(0)$, so

$$\begin{aligned} j_x|_{0+} - j_x|_{0-} &= \frac{\hbar}{2m}(\psi^*(0)\psi'(0_+) - \psi(0)\psi'^*(0_+) - \psi^*(0)\psi'(0_-) + \psi(0)\psi'^*(0_-)) & (5) \\ &= \frac{\hbar}{2m}(\psi^*(0)[\psi'(0_+) - \psi'(0_-)] - \psi(0)[\psi'^*(0_+) - \psi'^*(0_-)]) & (6) \\ &= \frac{\hbar}{2m}(\psi^*(0)[2\psi(0)/a] - \psi(0)[2\psi^*(0)/a]) & (7) \\ &= 0 & (8) \end{aligned}$$

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$$\begin{aligned} H &= \frac{(p - \frac{eA)^2}{c}}{2m} & (9) \\ &= \frac{p^2 - \frac{e}{c}(p \cdot A + A \cdot p) + \frac{e^2A^2}{c^2}}{2m} & (10) \\ & & (11) \end{aligned}$$

Choose $A = \frac{1}{2}B \times r$,and note that since $B = B\hat{z}$, we have $\nabla \cdot A = 0$ then

$$A \cdot p = \frac{1}{2}B \times r \cdot p = \frac{B}{2} \cdot (r \times p) = \frac{B}{2} \cdot L \tag{12}$$

$$p \cdot (A\psi) = -i\hbar\psi(\nabla \cdot A) + -i\hbar A \cdot (\nabla\psi) = -i\hbar A \cdot \nabla\psi = \frac{B}{2} \cdot L \tag{13}$$

$$A^2 = \frac{B}{4}(x^2 + y^2) \tag{14}$$

Combining these terms we find

$$H = \frac{p^2}{2m} + \frac{eBL_z}{2mc} + \frac{e^2B^2}{8mc^2}(x^2 + y^2) \tag{15}$$

The first term is spin-orbit coupling and the second relates to the Stark effect and magnetic susceptibility.

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a

$$[\Pi_x, \Pi_y] = \frac{e}{c}(-[p_x, A_y] + [p_y, A_x]) = \frac{ie\hbar}{c}(\nabla \times A) = \frac{ie\hbar}{c}B \tag{16}$$

$$\tag{17}$$

If we now define $X = \frac{c\Pi_y}{eB}$, then we have $[\Pi_y, X] = i\hbar$.

b

$$H = \frac{\Pi_x^2}{2m} + \frac{\Pi_y^2}{2m} + \frac{p_z^2}{2m} = \frac{\Pi_x^2}{2m} + \frac{e^2B^2X^2}{2mc^2} + \frac{p_z^2}{2m} \tag{18}$$

The new term is a SHO with $\omega = \frac{eB}{mc}$, so we can immediately write down

$$E = \frac{\hbar^2k^2}{2m} + \frac{eB}{2m}(n + \frac{1}{2}) \tag{19}$$