Lect 1: The Drude theory of metal

§ History of Condensend matter physics

1897 J.J. Tomoson — discovery of electron

1900 Drude — the 1st cond-matt physicist, — classic theory of electron

Sommerfeld — Quantum theory of electrons

Bloch — band theory of electrons in the lattice

Debye — lattice vibration — phonons

Onnes, Meissner — discovery of Superconductivity and Meissner effect

Kapitsa, Allen, Misener — discovery of Superfluidity of ^4He

Landau — set up the frame work of phase tranisition & interacting fermi systems

P.W. Anderson — many contributions to strongly correlated systems

Bednorz, Mueller — discovery of high Tc superconductor

Von Klitsing — discovery of QHE

Tsuei, et al — discovery of fractional QHE

Condensed matter physics is the largest branch of morden physics.

A physics of dirts ? ⟵ Pauli

More is different! ⟵ P. W. Anderson

§ Drude model

mobile electrons and immobile positive ions.
$\underbrace{\text{electrons}}_{\text{conduction}}$

typical density of electrons $\boxed{\begin{array}{l} n \sim 10^{22}\,\text{cm}^{-3} \sim 10^{23}\,\text{cm}^{-3} \\ R_s = \frac{r_s}{a_0} \sim 1 \sim 5 \end{array}}$ ← see table on page 5.

Basic assumptions

① Free electron approx: neglecting electron-ion interaction ← need to be abadined
independent electron approx: neglecting electron-electron interaction. ← very good
we do have collisions.

② An electron experiences one collision during a relaxation time τ, or mean-free time.

③ electrons achivel thermal equilibrium with their surrounding through collision. After each collision, electron looses memory of its previous velocity.

§ Electric conductivity

Suppose applying an electric fied E, then $\bar{v} = -\frac{eE}{m}\tau$

$$j = -ne\bar{v} = \frac{e^2 n\tau}{m}E \Rightarrow \boxed{\sigma = \frac{ne^2\tau}{m}}$$

or $\tau = \frac{m}{\rho n e^2}$, $\rho = 1/\sigma$. The typical order of ρ is $\sim 10^{-6}\,\Omega\cdot\text{cm}$

τ can be represented $\tau = \left(\frac{0.22}{\rho}\right)\left(\frac{r_s}{a_0}\right)^3 \times 10^{-14}$ sec, where ρ is measured in the unit of $10^{-6}\,\Omega\cdot\text{cm}$. τ is typically at $\boxed{10^{-14} \text{ to } 10^{-15}\text{ sec}}$ at room temperature.

mean free path $l = v_0 \tau$, v_0 is the average electron speed.

Boltzman distribution $\overline{v_0^2} = \frac{3k_BT}{m} \Rightarrow \sqrt{\overline{v_0^2}} \sim 10^7 \text{cm/s}$, $l \sim 1\sim10 \text{Å}$ } This is wrong.

due to Fermi statistics, $v_0 \sim v_f \sim 10^9 \text{cm/s}$, almost T-independent

τ is temperature dependent, which can be one order larger at low temperature.

l can reach 10^3 Å, … , about 10^3 lattice constants.

§ approximation ← relaxiation time

suppose there's an external force

$$p(t+dt) = \underbrace{\left(1 - \frac{dt}{\tau}\right)}_{\text{due to collision}} \left(p(t) + f(t)\,dt\right) = p(t) + f(t)dt - \frac{dt}{\tau}p(t)$$

$$\Rightarrow p(t+dt) - p(t) = \underbrace{-\frac{dt}{\tau}p(t)}_{\text{relaxiation}} + \underbrace{f(t)\,dt}_{\text{drift term}}$$

or
$$\boxed{\frac{dp(t)}{dt} = f(t) - \frac{p(t)}{\tau}}$$

AC conductivity: $E(t) = E(\omega)e^{-i\omega t}$

$$\frac{dp(t)}{dt} = -eE(\omega)e^{-i\omega t} - \frac{p(t)}{\tau} \qquad p(t) = p(\omega)e^{-i\omega t}$$

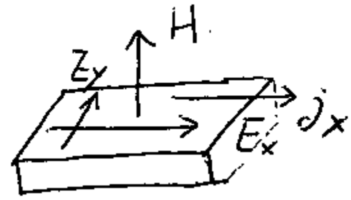
$$\Rightarrow -i\omega p(\omega) = -eE(\omega) - \frac{p(\omega)}{\tau} \Rightarrow p(\omega) = \frac{e}{i\omega - \frac{1}{\tau}}E(\omega)$$

$$j(\omega) = -\frac{en\,p(\omega)}{m} = \frac{ne^2/m}{1/\tau - i\omega} E(\omega) \Rightarrow$$

$$\boxed{\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}, \quad \text{where} \quad \sigma_0 = \frac{ne^2\tau}{m}.}$$

§ Hall effect & magnetoresistance

$$\rho(H) = \frac{E_x}{j_x}, \qquad R(H) = \frac{E_y}{j_x H}$$



$$\frac{d\vec{p}}{dt} = -e\left(\vec{E} + \frac{\vec{p}}{mc} \times \vec{H}\right) - \frac{\vec{p}}{\tau}$$

for steady currents, $\frac{d\vec{p}}{dt} = 0$, $\Rightarrow$

$$0 = -eE_x - \omega_c p_y - \frac{p_x}{\tau} \qquad \omega_c = \frac{eH}{mc}$$

$$0 = -eE_y + \omega_c p_x - \frac{p_y}{\tau}$$

$$\Rightarrow \quad 0 = \frac{ne^2\tau}{m} E_x - \omega_c\tau j_y - j_x \qquad \text{multiply } -\frac{ne\tau}{m}$$

$$0 = \frac{ne^2\tau}{m} E_y + \omega_c\tau j_x - j_y$$

$$\text{or} \quad \left.\begin{aligned} j_x + \omega_c\tau j_y &= \sigma_0 E_x \\ -\omega_c\tau j_x + j_y &= \sigma_0 E_y \end{aligned}\right\} \Rightarrow \quad \begin{aligned} j_x &= \frac{\sigma_0(E_x - \omega_c\tau E_y)}{1 + (\omega_c\tau)^2} \\ j_y &= \frac{\sigma_0(\omega_c\tau E_x + E_y)}{1 + (\omega_c\tau)^2} \end{aligned}$$

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \frac{\sigma_0}{1 + (\omega_c\tau)^2} \begin{bmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1 \end{bmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\text{or} \quad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{1}{\sigma_0} \begin{bmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{bmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$

Set $j_y = 0 \Rightarrow E_y = \frac{-\omega_c \tau}{\sigma_0} j_x \Rightarrow \boxed{R_H = -\frac{\omega_c \tau}{\sigma_0 H} = \frac{-e\tau}{mc} \cdot \frac{m}{ne^2\tau} = \frac{-1}{enc}}$

$E_x = \frac{1}{\sigma_0} j_x \Rightarrow \rho(H)$ doesn't depend on H, which is incorrect.

R_H depends on the density and charge of charge carriers.

§ local v.s. non local effect

The formula $j(r,\omega) = \sigma(\omega) E(r,\omega)$ is valid, when the wavelength λ of EM field is much larger the mean free path ℓ. E's effect mainly take place since the last collision, which occurs within ℓ.

§ Dielectric function

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \qquad \nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot E) - \nabla^2 E = -\frac{1}{c}\frac{\partial}{\partial t}(\nabla \times H) = -\frac{4\pi}{c^2}\frac{\partial}{\partial t}\vec{j} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\Rightarrow \quad -\nabla^2 E(r,\omega) = +\frac{4\pi}{c^2} i\omega j(r,\omega) + \frac{\omega^2}{c^2} E(r,\omega)$$

$$= \frac{\omega^2}{c^2}\left[1 + i\frac{4\pi\sigma(\omega)}{\omega}\right] E(r,\omega)$$

i.e. complex dielectric const for electron gas $\boxed{\varepsilon(\omega) = 1 + \frac{i 4\pi\sigma(\omega)}{\omega}}$.

or $D = E + 4\pi P$, $\quad j = +\frac{\partial}{\partial t}P = \sigma(\omega)E(\omega)$

$= \left(1 + i\frac{4\pi\sigma(\omega)}{\omega}\right) E \quad$ i.e. $-i\omega P(\omega) = \sigma(\omega) E(\omega)$

The zero of $\varepsilon(\omega)$ means an intrinsic mode of the system: $D \to 0$, but $E(\omega) \neq 0$.

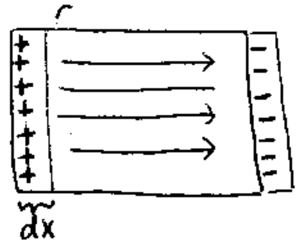
external field

plug in $\sigma(\omega) = \dfrac{e n^2 \tau / m}{1 - i\omega\tau} \xrightarrow[\omega\tau \gg 1]{\text{in the limit}} \dfrac{i e n^2}{m\omega}$

$$\Rightarrow \varepsilon(\omega) = 1 - \frac{4\pi e n^2}{m\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \quad \text{where } \omega_p^2 = \frac{4\pi e^2 n}{m}.$$

ω_p is the plasmon frequency.

Oscillation of ~~p~~ charged plasmons.



$E = 4\pi\sigma = 4\pi e n \cdot x$

$F = -QE = M\ddot{x}$

$\Rightarrow \ddot{x} = \dfrac{-Q \cdot 4\pi e n}{M} x = \underbrace{\boxed{\dfrac{e^2 4\pi n}{m}}}_{\omega_p^2} x$

① for $\omega < \omega_p$, $\varepsilon(\omega) < 0$.

the propagation wavevector is complex,

which is the forbidden region for E-M wave.

② for $\omega > \omega_p$, $1 > \varepsilon(\omega) > 0$. Metal is ~~transparent~~ for E&M wave.

The upper atmosphere also has similar dielectric behavior.

$$\nu_p = \frac{\omega_p}{2\pi} = 11.4 \times \left(\frac{r_s}{a_0}\right)^{-3/2} \times 10^{15}\ \text{Hz}$$

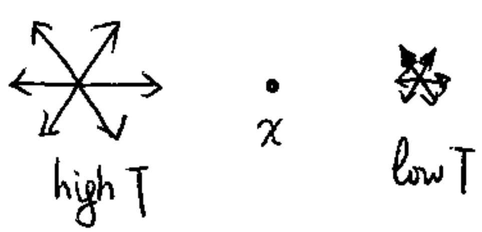
$$\lambda_p = 0.26 \left(\frac{r_s}{a_0}\right)^{3/2} \times 10^3\ \text{Å}$$

For alklai metals, they are transparent respect to ultra-violet.

§ Thermo conductivity

$$j^q = -K \nabla T, \quad \text{where } K \text{ is the thermo conductivity.}$$

1D:



At point x, the energy current forom left to right, comes from $x - v\tau$, and that from right to left comes from $x + v\tau$.

half of electrons

$$j^q = \frac{1}{2} n v \left[\mathcal{E}(T(x - v\tau)) - \mathcal{E}(T(x + v\tau)) \right]$$

$$= \frac{1}{2} n v \frac{d\mathcal{E}}{dT} \Delta T = \frac{1}{2} n v \frac{d\mathcal{E}}{dT} \frac{dT}{dx} (-2v\tau) = n v^2 \tau \frac{d\mathcal{E}}{dT} \left(-\frac{dT}{dx} \right)$$

For 3D, we need to replace v^2 with $v_x^2 = \frac{1}{3} v^2$.

$$n \frac{d\mathcal{E}}{dT} = \frac{N}{V} \frac{d\mathcal{E}}{dT} = \frac{1}{V} \frac{dE}{dT} = C_v \Rightarrow j^q = \frac{1}{3} v^2 \tau C_v (-\nabla T)$$

$$\Rightarrow \boxed{\chi = \frac{1}{3} v^2 \tau C_v = \frac{1}{3} l v C_v}$$

$$\sigma = \frac{n e^2 \tau}{m}$$

$$\Rightarrow \frac{\chi}{\sigma} = \frac{1}{3} \frac{v^2 C_v m}{n e^2} \xrightarrow{\text{Boltzman}} \begin{array}{l} \frac{1}{2} m v^2 = \frac{3 k_B T}{2} \\ C_v = \frac{3}{2} n k_B \end{array} \Rightarrow \boxed{\frac{\chi}{\sigma} = \frac{3}{2} \left(\frac{k_B}{e} \right)^2 T}$$

i.e $\frac{\chi}{\sigma T} = \frac{3}{2} \left(\frac{k_B}{e} \right)^2$ which agree with experiment. But the reasoning is wrong; two errors cancel each other, $\boxed{\begin{array}{l} C_v \simeq N_0 T k_B << \\ \frac{1}{2} m v^2 \simeq \frac{3}{5} E_F \end{array}}$.

<u>Wiedemann - Frantz law.</u>

§ Thermal power $\quad E = Q \nabla T$

1D: The mean velocity at point x, is an average from $L \to R$ and $R \to L$

$$v_Q = \frac{1}{2} \left[v(x - v\tau) - v(x + v\tau) \right] = -\tau v \frac{dv}{dx} = -\tau \frac{d}{dx} \frac{v^2}{2}$$

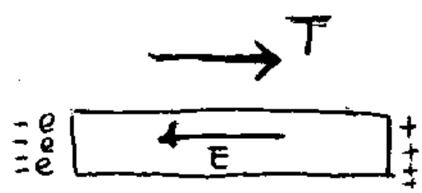
$3D \Rightarrow \vec{v}_Q = -\frac{\tau}{6}\frac{dv^2}{dT}(\vec{\nabla}T)$, set the x-axis along ∇T and $\overline{v_x^2} = \frac{1}{3}v^2$

This current should be balanced by the drift current

$$\vec{v}_E = -\frac{e\vec{E}\tau}{m}$$

$$\vec{v}_E + \vec{v}_Q = 0 \Rightarrow \frac{e\vec{E}\tau}{m} = -\frac{\tau}{6}\frac{dv^2}{dT}\vec{\nabla}T \quad \text{i.e.} \quad \vec{E} = -\frac{d(mv^2)}{6e\,dT}\nabla T = -\frac{k_B}{2e}\nabla T$$

$$\Rightarrow \boxed{Q = -\frac{k_B}{2e}}$$



more electron