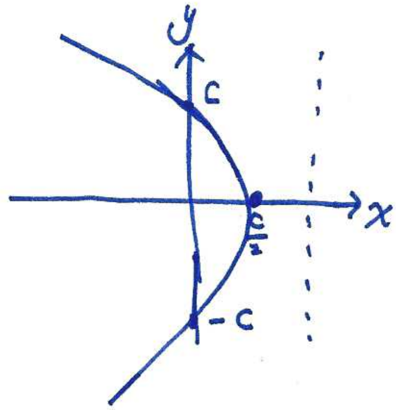
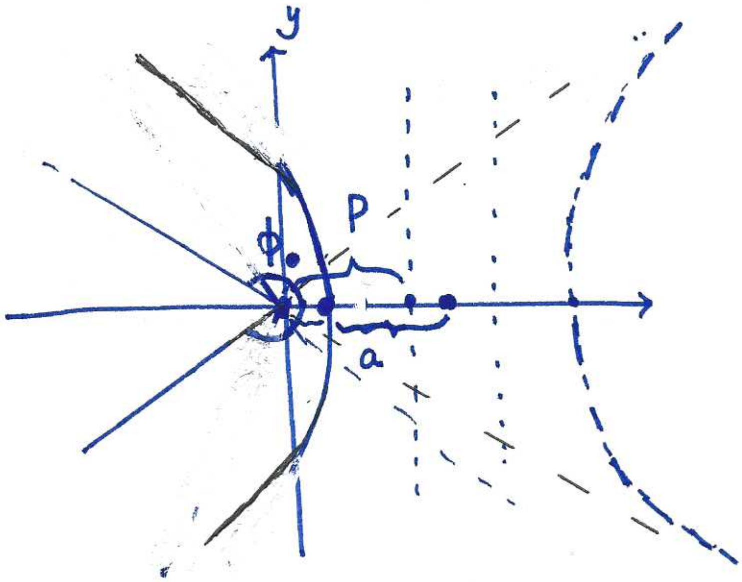
Lect 10 Kepler problem (II)

§ unbounded orbits : $r(\phi) = \dfrac{c}{1 + e\cos\phi}$



① $e = 1 \Rightarrow r(\phi = \pi) \to +\infty$, $y^2 = -2c\left[x - \frac{c}{2}\right]$

② $e > 1$:
$$\frac{\left(x - \frac{ec}{e^2-1}\right)^2}{\left(\frac{c}{e^2-1}\right)^2} - \frac{y^2}{\left(\frac{c}{\sqrt{e^2-1}}\right)^2} = 1$$



$P = c/e$

perihelion $\dfrac{c}{1+e}$

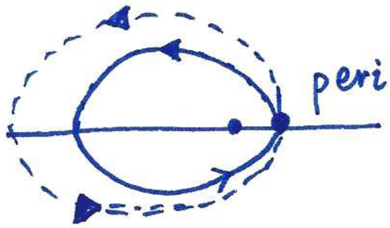
$a = \dfrac{c}{e^2-1}$

center $\left[\dfrac{ec}{e^2-1}, 0\right]$

define $\phi_0 = \cos^{-1} 1/e \Rightarrow$ r is finite when

$$\boxed{-(\pi - \cos^{-1}(1/e)) < \phi < \pi - \cos^{-1}(1/e)}$$

§ Change orbit



change from an elliptic orbit with (c_1, e_1) to another one with (c_2, e_2).

They ~~tagent~~ are at the prigee ⇒

$$\boxed{\frac{c_1}{1+e_1} = \frac{c_2}{1+e_2}}.$$

Define the thrust factor $\frac{v_2}{v_1} = \lambda$

$$\begin{cases} \lambda > 1 \Rightarrow \text{ forward thrust} \\ 1 > \lambda > 0 \quad \text{backward thrust} \end{cases}$$

$$\ell_2 = \lambda \ell_1$$

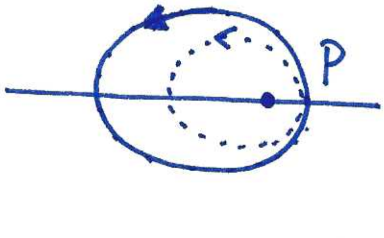
Since $c = \frac{\ell^2}{\mu \gamma}$

$$\Rightarrow \quad c_2 = \lambda^2 c_1 \qquad \Rightarrow \quad \frac{1+e_2}{1+e_1} = \frac{c_2}{c_1} = \lambda^2$$

or $\boxed{e_2 = \lambda^2 e_1 + (\lambda^2 - 1)}$

① If $\lambda > 1$, then $e_2 > e_1$. The two orbits have the same prigee the orbit becomes larger and more elliptical. At $e_2 \geq 1$, the orbit becomes open → parabola and hyperballa.

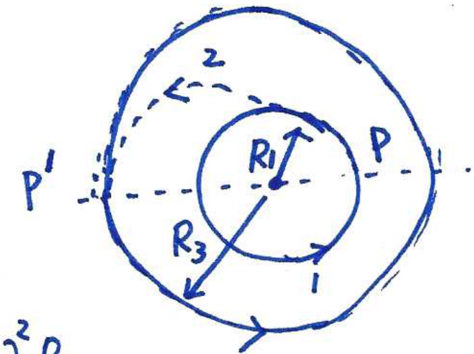
② If $\lambda < 1$, then $e_2 < e_1$. Then the new orbit becomes smallen and less elliptical. At $e_2 = 0$, the orbit becomes circluar.



how abont when $e_2 < 0$, then the equation of orbit changes to $r(\phi) = \frac{1}{1 - e_2 \cos\phi}$

Then the prigee and apogee switch.

Changing between circular orbits.



The eccentricity of the orbit is

$e_1 = 0$, and $C_1 = R_1$

The eccentricity of the orbit 2 is e_2

$$\begin{cases} r = \dfrac{C_2}{1+e_2\cos\phi} \\ C_2 = \lambda^2 R_1 \end{cases} \Rightarrow \frac{C_2}{1+e_2} = \frac{\lambda^2 R_1}{1+e_2} = R_1 \Rightarrow e_2 = \lambda^2 - 1$$

and the apogee $\dfrac{C_2}{1-e_2} = R_3 \Rightarrow C_2 = R_3(1-e_2)$

$$\lambda^2 R_1 = R_3(2-\lambda^2)$$

$$\Rightarrow \boxed{\lambda^2 = \frac{2R_3}{R_1+R_3}} \quad \text{or } \lambda = \sqrt{\frac{2R_3}{R_1+R_3}}$$

The 2nd thrust. $\rightarrow \begin{cases} r = C_3 = R_3 \\ e_3 = 0 \end{cases}$ $\quad C_3 = \lambda'^2 C_2$

$$\Rightarrow \lambda'^2 = \frac{C_3}{C_2} = \frac{R_3}{\lambda^2 R_1} = \frac{R_1+R_3}{2R_1} \quad \text{or} \quad \lambda' = \sqrt{\frac{R_1+R_3}{2R_1}}$$

The final speed and the initial speed

$$\begin{cases} v_3 = v_{2,app}\,\lambda' \\ \lambda v_1 = v_{2,peri} \end{cases} \text{ and } v_{2,app}\cdot R_3 = v_{2,peri} R_1$$

$$\Rightarrow v_3 = \lambda' \frac{v_{2,app}}{v_{2,peri}} \cdot \lambda v_1 = \lambda'\lambda \frac{R_1}{R_3} v_1 = \sqrt{\frac{R_1}{R_3}}\, v_1$$