Lecture 1 Basic concepts

§ Motivation — why to learn Classic Mechanics

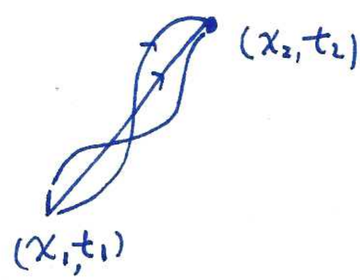
- Newtonian : $\vec{F} = m\vec{a}$ — convenient for simple problem (e.g. mass point)

Not good for quantum mechanics (QM). Force and acceleration are not well-defined in QM due to uncertainty principle.

- Lagrangian formalism: — least action principle

$$\mathcal{L}(x, \dot{x}) \rightarrow S = \int_{t_1}^{t_2} dt \; \mathcal{L}(x, \dot{x})$$

The classic paths satisfy $\delta S = 0$.



(c.f.) Fermat principle for light propagation.

geometric optics	$\longleftrightarrow$	classic mechanics
wave optics	$\longleftrightarrow$	QM (wave mechanics)

- Hamitonian — Canonical formalism

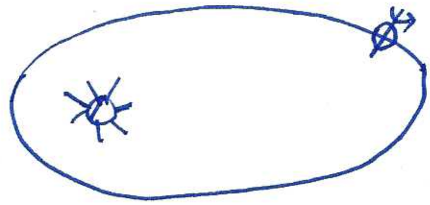
$$H(p, x) = p\dot{x} - \mathcal{L} \quad \Rightarrow \quad \begin{cases} \dot{p} = -\dfrac{\partial H}{\partial x} \\ \dot{x} = \dfrac{\partial H}{\partial p} \end{cases}$$

$xp - px$

→ Canonical quantization $\{x, p\}_p = 1 \longrightarrow [x, p] = i\hbar$ (matri... mecha...

What we will learn

- Galilean space-time and Newtonian mechanics
- Application of Newton's laws — projectile, harmonic oscillati
  circular motion.
- conservation laws — momentum / angular momentum,
  (integration of motions) energy
- Kepler problem



- Least action principle — Lagrange's equations

§ History:

Archimedes: Statics — Buoyancy, leverage, egulibrium

Aristotle: force causes motion — (wrong!)

Q: Why wasn't classic mechanics developed in Greek's time?

A: Lack of percisium measument of time.

(Mass, length are easy to measure, but time is difficult.).

§: Galilean's space-time

how to describle a simplest motion? say, a mass point, — a fly.

we need a coordinate system (x, y, z, t), We often denote

$\vec{r} = (x, y, z)$, and define velocity $\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} = (\dot{x}, \dot{y}, \dot{z})$.

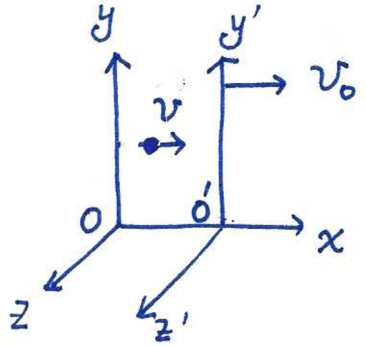
If we know $\vec{r}$ and $\vec{v}$ at an intial time, from our experience, it's motion is determined.

We have different choices of reference frames. How to connect these different descriptions?

Galiean ~~trans~~formation

$$\begin{cases} x = x' + v_0 t \\ y = y' \\ z = z' \\ t = t' \end{cases} \Rightarrow \frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \vec{v_0}$$

$$\text{or } \vec{v} = \vec{v}' + \vec{v_0}$$

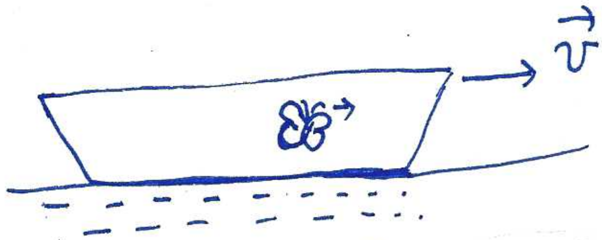


Q: Why don't we feel the motion of the earth?

A: The law of inertial — "Galiean's relativity"

☆ All physical laws should be the same in all the inertial frames.

Galileo's ship — 1632 thought experiment "Dialogue concerning two chief world systems".



This law is still correct for "Relativity", but is extended to include E & M laws, i.e. Maxwell equation.

Q: What's the inertial frame?

In the inertial frame, a particle remains at rest or moves with constant velocity $\vec{v}$ in the absence of forces, i.e. there's no interaction on it.

Newton's 1st law essentially defines the inertial frame. If frame F is inertial, and frame F′ has a relative velocity $\vec{v}$ with respect to F, then F′ is also an inertial frame.

$\boxed{\text{Inertial frames are an idealization}}$ — no rigorous inertial frames in reality.

But we do have good approximations, say, the earth.

Rigorously speaking, earth is not an inertial frame, — it moves around the earth, and spins. — $\boxed{\text{non-inertial frames.}}$

In non-inertial frames, the description of laws of motion need to be modified by adding inertial forces. If you want to understand tides, hurricans, Foucault pendulum, you need to use the fact that the earth frame is non-inertial!

☆ § Newton's 2nd law

$\vec{F} = m\vec{a}$, where m is the inertial mass, and $\vec{a} = \frac{d^2\vec{r}}{dt^2}$.

Since the 2nd order time derivative is used, it is very difficult to measure $\vec{a}$ in ancient time by using hourglasses. — This is why ancient Greeks didn't discover Newton's 2nd law.

- Precision measurement of time is based on the pendulum clock — Galileo.

- New technique stimulated the study of motions of nonuniform velocity — demand for new mathematics tools — calculus, and differential equations ← Newton invented new math!

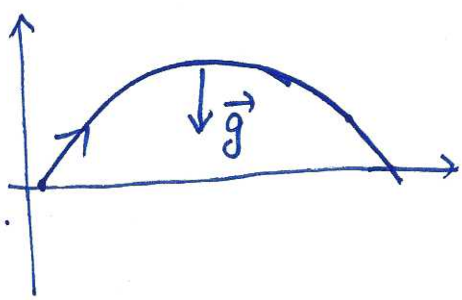
"Nature and nature's laws lay hid in night; ... Let Newton be and all was light" — Epitaph for Newton.

Simple example: free projectile

$$\ddot{\vec{r}} = \vec{g} \Rightarrow \frac{d^2}{dt^2}\vec{r} = \vec{g} \Rightarrow \dot{\vec{r}} = \int_{t_0}^{t} \vec{g}\, dt + \vec{v}_0$$

$$\vec{v} = \dot{\vec{r}} = \vec{g}(t-t_0) + \vec{v}_0 \Rightarrow \vec{r} = \int_{t_0}^{t} \dot{\vec{r}}\, dt + \vec{r}_0$$

$$\Rightarrow \boxed{\vec{r} = \vec{r}_0 + \vec{v}_0(t-t_0) + \frac{1}{2}\vec{g}(t-t_0)^2}$$



Newton's 2nd law defines "mass", which is invariant in non-relativistic physics. But it's no longer held in relativity in where mass depends on velocity. The mass at zero velocity is called rest mass m_0, and m_0 is a relativistic invariant. Their relation $m = \frac{m_0}{\sqrt{1-\beta^2}}$ and $\beta = \frac{v}{c}$

Newton's 2nd law — an equivalent form

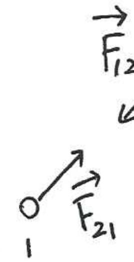
$$\vec{F} = \frac{d}{dt}\vec{p} \quad \text{with} \quad \vec{p} = m\vec{v}.$$

This frame is more convenient for motions of rockets, fluid, ect.

It remains correct in relativistic physics, but $\vec{p} = m\vec{v} = \frac{m_0\vec{v}}{\sqrt{1-\beta^2}}$.

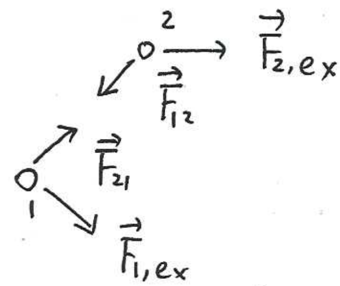
§ Newton's 3rd law

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$



central force

Consider a 2-particle system particle 1 and 2



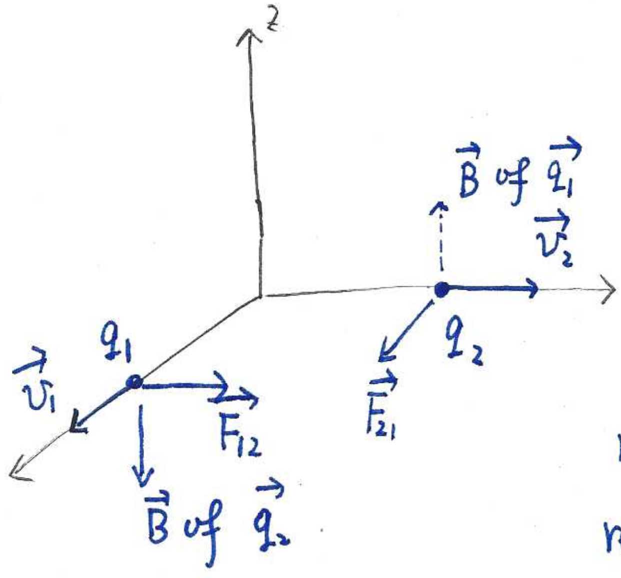
$$\begin{cases} \dfrac{d\vec{P}_1}{dt} = \vec{F}_{21} + \vec{F}_{2,ex} \\ \dfrac{d\vec{P}_1}{dt} = \vec{F}_{12} + \vec{F}_{1,ex} \end{cases} \Rightarrow \frac{d\vec{P}_{tot}}{dt} = \vec{F}_{ex,tot}$$

where $\vec{P}_{tot} = \vec{P}_1 + \vec{P}_2$, $\vec{F}_{ex,tot} = \vec{F}_{ex,1} + \vec{F}_{ex,2}$.

$$\boxed{\text{If } \vec{F}_{ex,tot} = 0 \Rightarrow \text{the total momentum conserved!}}$$

It can be easily generalized to multi-particle systems.

however, for E & M systems. Newton's 3rd law isn't held.



$\vec{F}_{12}$ and $\vec{F}_{21}$ non-colinear

how to rescure?

The E&M field carry momentum. The total momentum (including E&M) remain conserved!