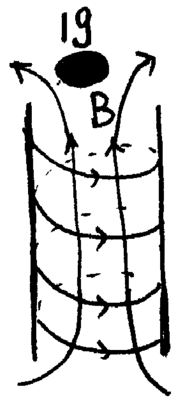
Lect 7 magnetic fielels in matter

§ Diamagnetism, paramagnetism, fenomagnetism



$B_z = 18{,}000$ Gauss, at center $\quad \frac{dB_z}{dz} = 1700$ G/cm

Forces are proportional to $\frac{dB_z}{dz}$, thus are largest at the edge of coils.

1g ~ 1000 dyn

(−: up ; + down)

	1 gram	F	
Diamagnet:	H_2O	−22 dyn	
	Cu	−2.6	repel to outside of the coil
	$\boxed{\text{graphite} \quad -110}$		
	liquid N_2	−10 (78K)	
	diamond	−16	
	Pb	−37	
paramgnets	liquid O_2	+7,500 (90K)	
	$NiSO_4$	+830	pull into the coil
	Na	20	
Ferromagnet	Fe	+400,000	
	Fe_3O_4	+120,000	

difference between fenomagnet and diamgnet / parament

if we reduce the current of coil to half, then forces for diamagnet / parament ⟶ 1/4, but forces on fenomagnets ⟶ 1/2.

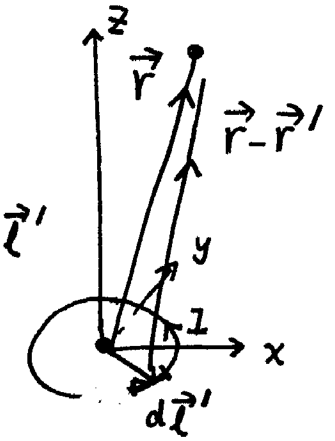
proportional to the squre of field strength

proportional to field itself

§ magnetic dipole

$d\vec{l}'$ along current direction

$$\vec{A}(\vec{r}) = \frac{I}{c}\oint \frac{d\vec{l}'}{|\vec{r}-\vec{r}'|} \xrightarrow[\text{expansion}]{\text{monopole}} \frac{\vec{m}\times\hat{r}}{r^2}$$

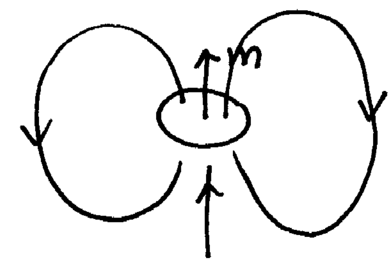


$$\text{where } \vec{m} = \frac{I}{2c}\oint \vec{r}'\times d\vec{l}'$$

refer to lecture 2

$$= \frac{I\vec{a}}{c}$$

$$\vec{B}(r) = \nabla\times\left(\frac{\vec{m}\times\hat{r}}{r^2}\right) = \frac{3(\vec{m}\cdot\hat{r})\,\hat{r}-\vec{m}}{r^3}$$



Set $\vec{m}$ along the z-axis

$$A_x = -\frac{my}{r^3}, \quad A_y = \frac{mx}{r^3}, \quad A_z = 0$$

$$B_x = \frac{3mzx}{r^5}, \quad B_y = \frac{3myz}{r^5}, \quad B_z = \frac{3z^2-r^2}{r^5}\,m$$

$$= \frac{m}{r^3}(3\cos^2\theta - 1)$$

$$= \frac{3m}{r^3}\sin\theta\cos\theta\cos\varphi \qquad = \frac{3m}{r^3}\sin\theta\cos\theta\sin\varphi$$

c.f electric dipole

$$\varphi(\vec{r}) = \int\frac{\rho\, dv'}{|\vec{r}-\vec{r}'|} \xrightarrow{\text{diapole}} \frac{1}{r^2}\int \hat{r}\cdot\vec{r}'\,\rho\, dv' = \frac{\hat{r}\cdot\vec{P}}{r^2}$$

$$\vec{E}(\vec{r}) = -\nabla\varphi(\vec{r}) = -\nabla\left(\vec{P}\cdot\frac{\vec{r}}{r^3}\right) =$$

$$= -(\vec{P}\cdot\vec{r})\,\nabla\frac{1}{r^3} - \frac{\nabla(\vec{P}\cdot\vec{r})}{r^3} = \frac{3\hat{r}}{r^5}(\vec{P}\cdot\vec{r}) - \frac{\vec{P}}{r^3} = \frac{3(\vec{P}\cdot\hat{r})\hat{r}-\vec{P}}{r^3}$$

E-field of an electric dipole is the same as the B-field of a magnetic dipole

Forces on a magnetic diapole, suppose the dipole is put at $r=0$

$$\vec{F} = \frac{I}{c}\oint d\vec{l}'\times\vec{B}(\vec{r}') \quad = 0 \text{ if for uniform magnetic field.}$$

expand to 1rst order $\Rightarrow$ $B_\mu(\vec{r}') = r'_\nu \partial_\nu B_\mu(\vec{r}') + B_\mu(\vec{r}'=0)$

$$\Rightarrow \vec{F} = \frac{I}{c}\oint d\vec{l}'\times(\vec{r}'\cdot\vec{\nabla})\vec{B}_\mu(\vec{r})\Big|_{\vec{r}=0}, \text{ or}$$

$$F_i = \frac{I}{c}\,\epsilon_{ijk}\Big\{\oint r'_l\, dl'_j\Big\}\,[\nabla_{,l}\, B_k(\vec{r}=0)]\Big|_{\vec{r}=0}$$

using $\boxed{\oint(\vec{c}\cdot\vec{r})d\vec{l} = \vec{a}\times\vec{c}}$

For fixed k, $\nabla\, B_k(\vec{r}=0)$ is a const vector

$$\oint dl'_j\; r'_l\cdot\nabla_{0l}\, B_k(\vec{r}=0) = \Big[\oint d\vec{l}\,(\vec{r}\cdot\nabla\, B_k(\vec{r})\Big|_{\vec{r}=0})\Big]_j = \Big[\vec{a}\times\nabla\, B_k(\vec{r})\Big|_{\vec{r}=0}\Big]_j$$

$$\Rightarrow F_i = \epsilon_{ijk}\,\epsilon_{jln}\,\vec{m}_l\,\nabla_n B_k(\vec{r})\Big|_{\vec{r}=0}$$

$$= (\delta_{kl}\delta_{in} - \delta_{kn}\delta_{li})\; m_l\, \nabla_{0,n} B_k(\vec{r})\Big|_{\vec{r}=0}$$

$$= m_l\,\nabla_{,i}\, B_l(\vec{r})\Big| - m_i\,(\nabla\cdot\vec{B})\Big|_{\vec{r}=0}$$

$$\Rightarrow F_i(\vec{r}) = m_l\,\nabla_i\, B_l(\vec{r}), \quad \text{for const } \vec{m}$$

$$\Rightarrow \vec{F} = \nabla(\vec{m}\cdot\vec{B})$$

c.f electric dipole:

assume $Q=\int\rho dv = 0$, and charge is close to $r=0$

$$\vec{F} = \int\rho dv\,\vec{E}(r'), \qquad E_\mu(r') = E(r'=0) + r'_\nu\partial_\nu E_\mu(\vec{r}')\Big|_{r'=0}$$

$\Rightarrow$

$$F_i = \int \rho(r')\, d^3\vec{r}'\ r_j'\, \partial_j E_i(\vec{r}')\Big|_{r'=0} = \left[\int r_j' \rho(r')\, d^3\vec{r}'\right] \partial_j E_i(\vec{r}')\Big|_{r'=0}$$

$$= P_j \nabla_j E_i \quad \Rightarrow \quad \vec{F} = (\vec{P}\cdot\vec{\nabla})\vec{E}$$

For <u>electro static case</u>, and $\vec{P}$ is a const vector, $(\vec{P}\cdot\vec{\nabla})\vec{E}$ is equivilate to

$$\vec{\nabla}(\vec{P}\cdot\vec{E}) = \vec{P}\times\nabla\times\vec{E} + (\vec{P}\cdot\vec{\nabla})\vec{E} = (\vec{P}\cdot\vec{\nabla})\vec{E} \quad \Rightarrow \quad \vec{F} = \nabla(\vec{P}\cdot\vec{E}).$$

§ torque on dipoles

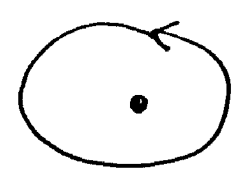
mag-dipole $\vec{N} = \oint \vec{r}'\times d\vec{F} = \frac{I}{c}\oint \vec{r}'\times(d\vec{l}'\times\vec{B}) = \frac{I}{c}\oint (\vec{r}'\cdot\vec{B})\,d\vec{l}' - (\vec{r}'\cdot d\vec{l}')\vec{B}$

$$= \vec{a}\times\frac{I}{c}\vec{B} - \frac{I}{c}\Big[\underbrace{\oiint \nabla\times\vec{r}'}_{=0}\Big]\vec{B} = \vec{m}\times\vec{B}$$

c.f.

electric dipole $\vec{N} = \int \vec{r}'\times d\vec{F} = \underbrace{\iiint \vec{r}'\rho\, dv}\times\vec{E} = \vec{P}\times\vec{E}$

§ dia-magnetism : general pheonomenon



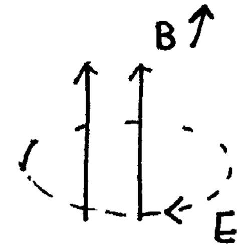
$$I = \frac{-ev}{2\pi r} \Rightarrow \vec{m} = \frac{I}{c}\pi r^2 = \frac{-evr}{2c}$$

$$L = m_e v r$$

$$\Big\} \Rightarrow \vec{m} = -\frac{e}{2m_e c}\vec{L}$$

if we increase external magnetic field

$$\frac{d\Phi}{dt} = \pi r^2\frac{dB}{dt} = -\frac{1}{c}\oint \vec{E}\cdot d\vec{l} = -\frac{2\pi r E}{c} \Rightarrow E = -\frac{r}{2c}\frac{dB}{dt}$$



$$m_e \frac{dv}{dt} = -eE = \frac{er}{2c}\frac{dB}{dt}$$

$$\Rightarrow dv = \frac{er}{2m_e c}\, dB$$

$$d\vec{m} = -\frac{er}{2c}\, dv = -\frac{e^2 r^2}{4 m_e c^2}\, d\vec{B} \longrightarrow \chi = -\frac{e^2 r^2}{4 m_e c^2}$$

Landau diamagnetism

For: per gram matter, it has $\frac{6.02\times10^{23}}{2} \approx 3\times10^{23}$ electron,

$1\,\text{mol}\ \boxed{1p+1n+1e} = 2\,\text{gram}$

$r \approx 0.5\,\text{Å}$, let take $\Delta B = 18{,}000$ Gauss $= 1.8\,T$

$$\Rightarrow n\cdot\Delta m = \frac{n e^2 r^2}{4 m_e c^2}\Delta B \approx 0.95\times10^{-2}$$

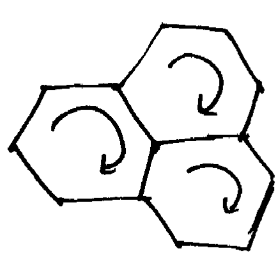
$$\text{Force}: \ n\cdot\Delta m \underbrace{\frac{\partial B}{\partial z}}_{1700\,G/cm} \approx 16\ \text{dyn}$$

$\boxed{1\ \text{Newton} = 10^5\ \text{dyn}}$

$$\Delta v = \frac{er}{2m_e c}\Delta B \approx 10^3\ \text{cm/s} \approx 10\ \text{m/s} \ll \text{Bohr velocity} = \frac{c}{137}.$$

diag-magnetism is not very strong! which is often diminated over by paramagnetism.

Graphite / graphene / Benzene has large diamagnetism due to delocalized electron orbits.



§ Paramagnetism — electron spin

$\vec{L} = \frac{\hbar}{2}$

it's not clean the origin of spin-magnetic moment.

$\vec{m} = \frac{e}{mec}\,\frac{\hbar}{2} = 1$ Bohr magneton

$= 1 \times 10^{-19}$ erg/G

$$\vec{m}_s = -\frac{ge}{2mec}\vec{L} \quad \text{where } g = 2.$$

Lande factor

if all the electrons spin are aligned with the B-field, what's the force?

1 gram ~ 3×10^{23} electron, — total spin moments

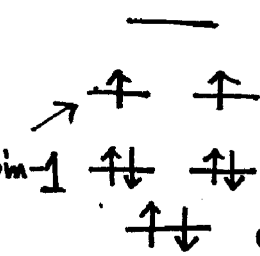
$3 \times 10^{23} \times 10^{-19} = 3000$ erg/G

$\frac{\partial B}{\partial z} \sim 1700$ G/cm

$\Rightarrow F \sim 5 \times 10^{6}$ dyn ~ 50 N!

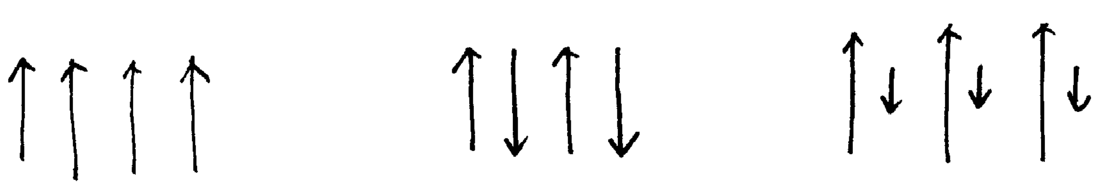
no. Not all of electrons align their spin momentum. electron spin can ↑ and ↓, and mostly canceled. In ferromagnet, F can reach 1/10 of this force.

example O_2 $1s^2 2s^2 2p^4$ (Hund's rule)



NO ~ 15 electron - paramagnet

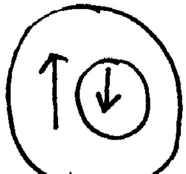
§ Ferromagnetism, anti-ferromagnetism, ferri-magnetism



many transition metal oxides.

Fe_3O_4

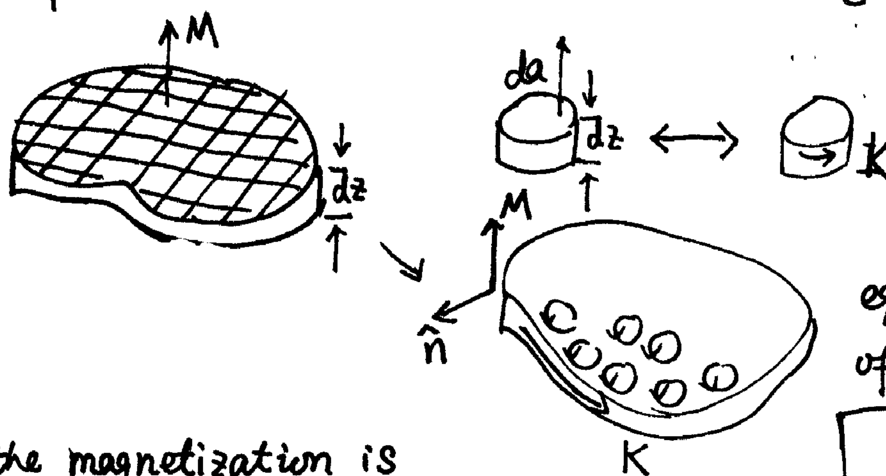
Fe
Co
Ni



Lect 8 magnetic fields in matter (II)

§ Magnetization: M, magnetic dipole moment per unit volume.

Bound current picture: imagine that we have a uniformly distributed M with a boundry. We can divide the area into many small pieces.
Each of them has dipole moment $M\,da\,dz$ $\quad \frac{K}{c} dz\,da = M\,dz\,da$

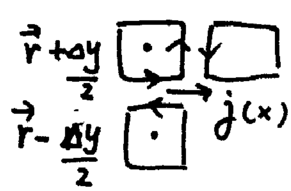


equivalent to a belt of surface current

$$\boxed{M = K/c \quad \text{or} \quad \vec{K} = c\vec{M}\times\hat{n}}$$

If the magnetization is non-uniform, it will induce the bulk bound current.

If M is along $\hat{z}$-direction,



$$j_x(\vec{r}) \cdot \Delta y \cdot \Delta z = c\left[M_z(\vec{r}+\tfrac{\Delta y}{2}) - M_z(\vec{r}-\tfrac{\Delta y}{2})\right]\Delta z$$

$$j_x(\vec{r}) = c\,\frac{\partial M_z}{\partial y}.$$

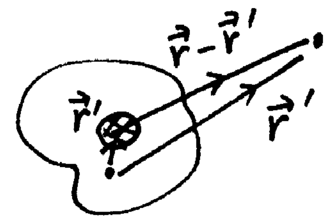
if M is along $\hat{y}$-direction, we will have $j_x(\vec{r}) = -c\,\frac{\partial M_y}{\partial z}$.

Combine them together $\Rightarrow$ $j_x(\vec{r}) = c\left(\frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}\right) = c\,(\nabla\times M)_x$

or $\boxed{\vec{j}(\vec{r}) = c\,\nabla\times\vec{M}}$

A more formal derivation: the vecter potential of magnetic dipole.

$$A(\vec{r}) = \frac{\vec{m}\times\hat{r}}{r^2}.$$ Imagine we have a spatial distribution of M, such at $d\vec{m} = \vec{M}\,dv$



$$\Rightarrow A(\vec{r}) = \int \frac{M(\vec{r}')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, d^3\vec{r}'$$

$$= \int\left[M(\vec{r}')\times\nabla_{r'}\frac{1}{|\vec{r}-\vec{r}'|}\right] d^3\vec{r}'$$

using $\nabla\times[\vec{f}\, g] = (\nabla\times\vec{f})\, g - \vec{f}\times\nabla g$

$$= -\int \nabla_{r'}\times\left[\frac{M(\vec{r}')}{|\vec{r}-\vec{r}'|}\right] d^3\vec{r}' + \int \frac{\nabla_{r'}\times M(\vec{r}')}{|\vec{r}-\vec{r}'|}\, d^3\vec{r}'$$

using $\int_V (\nabla\times\vec{f})\, dv = -\oint_S \vec{f}\times d\vec{a}$

Prob 1.60B

$$= \int \frac{\nabla_{r'}\times M(\vec{r}')}{|\vec{r}-\vec{r}'|}\, d^3\vec{r}' + \oint_S \frac{M(\vec{r}')\times d\vec{a}'}{|\vec{r}-\vec{r}'|}$$

$$\rightarrow \frac{1}{c}\int \frac{j(\vec{r}')\, d^3\vec{r}'}{|\vec{r}-\vec{r}'|} + \frac{1}{c}\oint_S \frac{\vec{K}(\vec{r}')\, da'}{|\vec{r}-\vec{r}'|}$$

$$\Rightarrow \boxed{j(\vec{r}) = c\,\nabla\times M(r), \qquad \vec{K} = c\, M(\vec{r}')\times\hat{n}}$$

bulk current   surface current

c.f. polarization charge

single dipole   an volume of charge dipol

$$\varphi(\vec{r}) = \frac{\hat{r}\cdot\vec{p}}{r^2} \longrightarrow \varphi(\vec{r}) = \int \frac{\vec{p}(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, d^3\vec{r}'$$

$$\Rightarrow \varphi(\vec{r}) = \int \vec{p}(\vec{r}')\cdot\nabla_{r'}\frac{1}{|\vec{r}-\vec{r}'|}\, d^3\vec{r}' = \int\left[\nabla_{r'}\cdot\left(\frac{\vec{p}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right) - \frac{\nabla\cdot\vec{p}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right] d^3\vec{r}$$

$$= \int \frac{-\nabla\cdot\vec{P}(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3\vec{r}' + \oint \frac{\vec{P}(\vec{r}')\cdot d\vec{a}}{|\vec{r}-\vec{r}'|}$$

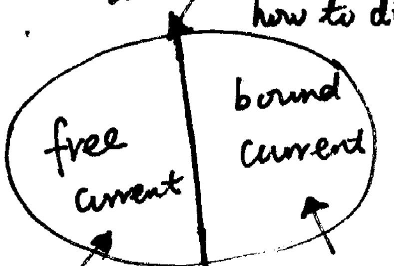
$$\rightarrow \int \frac{\rho(r')}{|\vec{r}-\vec{r}'|} d^3\vec{r}' + \oint \frac{(\vec{P}(\vec{r}')\cdot\hat{n})\, da}{|\vec{r}-\vec{r}'|}$$

$$\Rightarrow \boxed{\begin{aligned} \rho(r) &= -\nabla\vec{P} \\ \sigma(r) &= \vec{P}\cdot\hat{n} \end{aligned}}$$

§ Magnetic field strength:

We would like to distinguish $\overbrace{\text{the currents that we can easily controll}}^{\text{free currents}}$, and $\underbrace{\text{those we cannot}}_{\text{bound current}}$. We want in the Maxwell equations only free current appear, and absorb the bound current into media properties.

it is not unique how to divide.



free to controll

difficult to controll

$$J_{bound} = c\,\nabla\times\vec{M}$$

Let us first only consider steady situation.

$$\nabla\times\vec{B} = \frac{4\pi}{c}(\vec{j}_{free} + \vec{j}_{bound}) = \frac{4\pi}{c}\vec{j}_{free} + 4\pi\nabla\times\vec{M}$$

$$\Rightarrow \nabla\times(\vec{B}-4\pi\vec{M}) = \frac{4\pi}{c}\vec{j}_{free},$$

define $\vec{H} = \vec{B} - 4\pi\vec{M}$ $\Rightarrow$ $\boxed{\nabla\times\vec{H} = \frac{4\pi}{c}\vec{j}_{free}}$

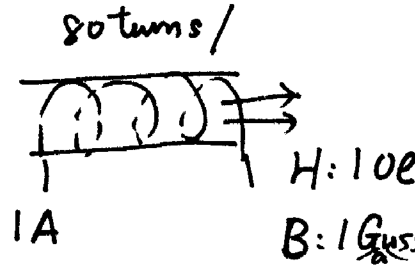
$$\oint \vec{H}\cdot d\vec{l} = \frac{4\pi}{c} I_{free}$$

c.f. $\vec{E}$ and $\vec{D}$. $\nabla\cdot\vec{E} = 4\pi\rho = 4\pi(\rho_{free} + \rho_{bound}) = 4\pi\rho_{free} - 4\pi\nabla\cdot\vec{P}$

define $\vec{D} = \vec{E} + 4\pi\vec{P}$ $\Rightarrow$ $\boxed{\nabla\cdot\vec{D} = 4\pi\rho_{free}}$

Practically, $\vec{H}$ is more useful than $\vec{D}$. We can cancoll current easily, but not charge. On the other hand, $\vec{E}$ is easy to control and measure, but not $\vec{B}$.

$\boxed{H: (Oe).\ B\,(G).}$ 1 oersted $\Leftrightarrow$ 1 Gauss in the vaccum.



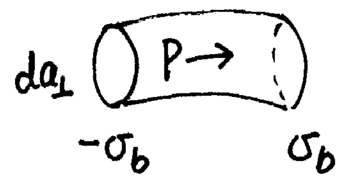
magnetic susceptibility $\quad M = \chi_m H$

$$B = H + 4\pi M = H(\underbrace{1 + 4\pi\chi_m}_{\text{permeability}}) = \mu H \Rightarrow \mu = 1 + 4\pi\chi_m$$

c.f. $$D = E + 4\pi P = E(\underbrace{1 + 4\pi\chi_e}_{\text{permittivity}}) = \varepsilon E \Rightarrow \varepsilon = 1 + 4\pi\chi_e$$

if allow time-dependence, the bound charge can also contribute to current

Conside a small segment parallel to $\vec{P}$,



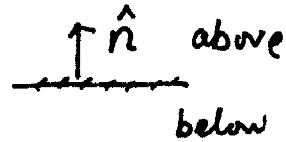
as $\vec{P}$ increas, the ~~current~~ charge at the cross section changes

$$dI = \frac{\partial \sigma_b}{\partial t} da_\perp = \frac{\partial P}{\partial t} da_\perp \Rightarrow \vec{j}_{bound} = \frac{\partial \vec{P}}{\partial t}$$

$$\Rightarrow \nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{j} = j_{free} + \nabla \times M + \frac{\partial \vec{P}}{\partial t}$$

$$\Rightarrow \nabla \times (\vec{B} - 4\pi\vec{M}) = 4\pi j_f + \frac{1}{c}\frac{\partial}{\partial t}[\vec{E} + 4\pi\vec{P}] \Rightarrow \boxed{\nabla \times \vec{H} = 4\pi\vec{j}_f + \frac{\partial \vec{D}}{c\,\partial t}}$$

§ Boundary conditions



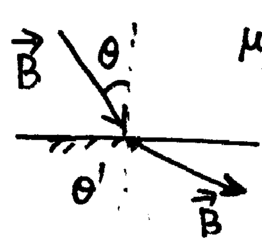
$$\nabla\cdot\vec{H} = -\nabla\cdot\vec{M} \;\Rightarrow\; H^{\perp}_{above} - H^{\perp}_{below} = -\left(M^{\perp}_{above} - M^{\perp}_{below}\right)$$

$$\nabla\times\vec{H} = \frac{4\pi}{c}\vec{K}_f\times\hat{n} \;\Rightarrow\; H^{\parallel}_{above} - H^{\parallel}_{below} = \frac{4\pi}{c}\,\vec{K}_f\times\hat{n}\,.$$

or in terms of $\vec{B}$ $\Rightarrow$ $B^{\perp}_{above} - B^{\perp}_{below} = 0$ $\quad \nabla\cdot\vec{B} = 0$

$$B^{\parallel}_{above} - B^{\parallel}_{below} = \frac{4\pi}{c}\,\vec{K}\times\hat{n}\,.$$

For a permanent magnet with no free current, and permeability μ.

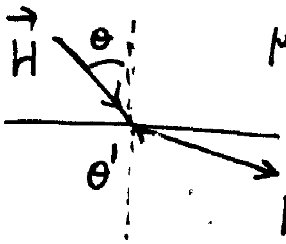


$\mu_{out} = 1$, $\vec{B} = H$

μ_{in} $\vec{B} = \mu H_{in}$

$$\boxed{\begin{aligned} B^{out}_{\perp} &= B^{in}_{\perp} \\ H^{out}_{\parallel} = B^{out}_{\parallel} &= H^{in}_{\parallel} = \frac{B^{in}}{\mu_{in}} \end{aligned}}$$

$$\frac{\tan\theta'}{\tan\theta} = \frac{B^{in}_{\parallel}}{B^{out}_{\parallel}} = \mu_{in}$$



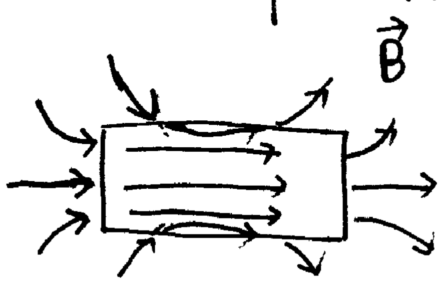
$\mu = 1$ $\quad \vec{B} = \vec{H}$

μ_{in} $\quad \vec{B} = \mu_{in} H$

$$\frac{\tan\theta'}{\tan\theta} = \mu_{in}\,.$$ in terms of H lines.

Consider a permanent magnet with $\mu_{in} \gg 1$, with $\vec{M}$ const inside

$\vec{B}$ remains finite inside $\Rightarrow$ $\vec{H} = 0$ inside



$$\left.\begin{aligned} \vec{B} &\approx 4\pi\vec{M}, \\ \vec{H} &\approx 0 \end{aligned}\right\} \text{inside}\,,$$