Problem 2.36

(a) $\boxed{\sigma_a = -\dfrac{q_a}{4\pi a^2};}$ $\boxed{\sigma_b = -\dfrac{q_b}{4\pi b^2};}$ $\boxed{\sigma_R = \dfrac{q_a + q_b}{4\pi R^2}.}$

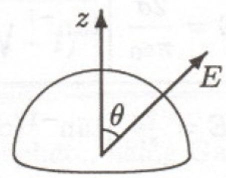
(b) $\boxed{\mathbf{E}_{\text{out}} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_a + q_b}{r^2}\,\hat{\mathbf{r}},}$ where $\mathbf{r}$ = vector from center of large sphere.

(c) $\boxed{\mathbf{E}_a = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_a}{r_a^2}\,\hat{\mathbf{r}}_a, \quad \mathbf{E}_b = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_b}{r_b^2}\,\hat{\mathbf{r}}_b,}$ where $\mathbf{r}_a$ ($\mathbf{r}_b$) is the vector from center of cavity a (b).

(d) $\boxed{\text{Zero.}}$

(e) σ_R changes (but not σ_a or σ_b); $\mathbf{E}_{\text{outside}}$ changes (but not $\mathbf{E}_a$ or $\mathbf{E}_b$); force on q_a and q_b still zero.

Problem 2.38



Inside, $\mathbf{E} = 0$; outside, $\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$; so

$\mathbf{E}_{\text{ave}} = \frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\,\hat{\mathbf{r}}$; $f_z = \sigma(E_{\text{ave}})_z$; $\sigma = \frac{Q}{4\pi R^2}$.

$$F_z = \int f_z da = \int \left(\frac{Q}{4\pi R^2}\right)\frac{1}{2}\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\right)\cos\theta\, R^2 \sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{2\epsilon_0}\left(\frac{Q}{4\pi R}\right)^2 2\pi \int_0^{\pi/2} \sin\theta\cos\theta\, d\theta = \frac{1}{\pi\epsilon_0}\left(\frac{Q}{4R}\right)^2 \left(\tfrac{1}{2}\sin^2\theta\right)\Big|_0^{\pi/2} = \frac{1}{2\pi\epsilon_0}\left(\frac{Q}{4R}\right)^2 = \boxed{\frac{Q^2}{32\pi R^2\epsilon_0}.}$$

Problem 2.40

(a) W = (force)×(distance) = (pressure)×(area)×(distance) = $\boxed{\dfrac{\epsilon_0}{2}E^2 A\epsilon.}$

(b) W = (energy per unit volume)×(decrease in volume) = $\left(\epsilon_0\frac{E^2}{2}\right)(A\epsilon)$. Same as (a), confirming that the energy lost is equal to the work done.

Problem 3.1

The argument is exactly the same as in Sect. 3.1.4, except that since $z < R$, $\sqrt{z^2 + R^2 - 2zR} = (R - z)$, instead of $(z - R)$. Hence $V_{\text{ave}} = \dfrac{q}{4\pi\epsilon_0}\dfrac{1}{2zR}[(z + R) - (R - z)] = \boxed{\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{R}.}$ If there is more than one charge inside the sphere, the average potential due to interior charges is $\dfrac{1}{4\pi\epsilon_0}\dfrac{Q_{\text{enc}}}{R}$, and the average due to exterior charges is V_{center}, so $V_{\text{ave}} = V_{\text{center}} + \frac{Q_{\text{enc}}}{4\pi\epsilon_0 R}$. ✓

Problem 3.2

A stable equilibrium is a point of local minimum in the potential energy. Here the potential energy is qV. But we know that Laplace's equation allows no local minima for V. What *looks* like a minimum, in the figure, must in fact be a saddle point, and the box "leaks" through the center of each face.