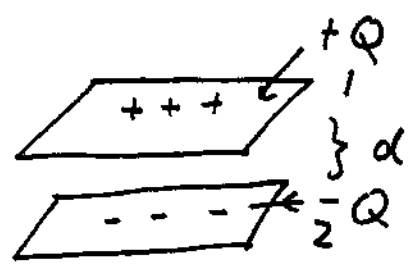# Lect 17    Dielectrics

Consider a capacitor $C = \dfrac{Q}{V_{12}} = \dfrac{A}{4\pi d}$.

if we insert insulating material, we will

find $V_{12}$ is weakened, and thus $C$ increases. The difference

can be summerize by a constant — $\varepsilon$ (dielectric constant).

$$\boxed{E_{in\text{-}media} = \dfrac{E_{vaccum}}{\varepsilon}} \Rightarrow C_{with\text{-}media} = \varepsilon \, C_{vaccum}.$$

dielectric constant for common materials

| | | | |
|---|---|---|---|
| $H_2O$ | 80, | S | 40 |
| Benine | 2.28 | $NH_3$ | 22 |
| NaCl | 6.12. | rubber | ~4 |
| | | wax | 2.1 ~ 2.5 |

we need to construct a theory to describle the effect of E-field
in the medium. In the dielectric material, there's no free charge,
thus the external electric field cannot be perfectly screened as
in metal. Nevertheless, electric-polarization will be induced which
partially cancel the external field.

§ diapole of atom & and molecules

atom

$-q$, uniformally
distribute
around nucleus.
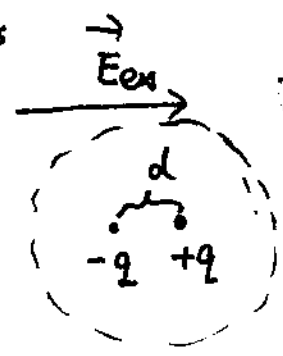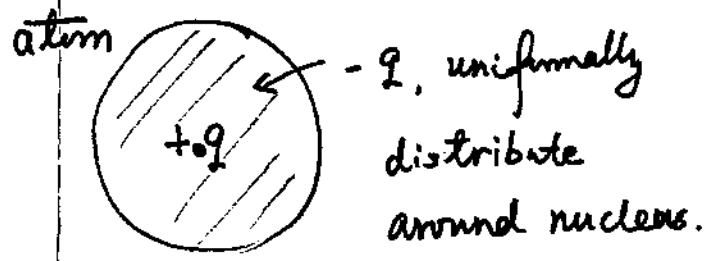
assume the the atom raidus $a$, and charge uniformally distributed

$\Rightarrow$ now nucleus at a distance "d" from the center

$\Rightarrow$ E at distance $d$ is $\frac{q^e}{a^2} \cdot \frac{d}{a}$, which should equal
to $E_{ex}$

$\Rightarrow$ $\frac{qd}{a^3} = E_{ex}$ $\Rightarrow$ dipole $p = qd = a^3 E_{ex} = \frac{3}{4\pi} Vol \cdot E_{ex}$

define polarizbility $\alpha$ $\Rightarrow$ $p = \alpha E$, where $\alpha \approx a^3$.

$\alpha$ carries the unit of volume. If we express $\frac{\alpha}{a_0^3}$ which $a_0$ is Bohr radius

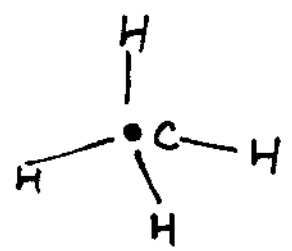$\Rightarrow$ $\frac{\alpha}{a_0^3} = $ 0.66 (H), 0.2(He), 24.3 (Li), 43.4 (K)

59.6 (Cs), 1.76 (C), 24.1 (Na), 0.396 (Ne), 1.64(Ar)

how about molecule:

molecule has a concrete shape: for molecules with high
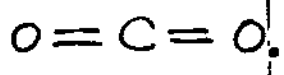symmetries, say $CH_4$, the induced dipole $\leftarrow$ no-longer spherical
is parallel to external field

$$\vec{p} = \alpha \vec{E}_{ex}.$$

However, this is not always the case, say, $CO_2$.

The responses for the field along the bond $\quad O = C = O$.
or perpendicular to the bond is quite different! $\Rightarrow$

$$\vec{p} = \alpha_\perp \vec{E}_\perp + \alpha_{//} \vec{E}_{//}. \quad \text{thus} \quad \vec{P} \nparallel \vec{E}.$$

generally

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

↑ polarizibility tensor.

*Later on we can prove that $\alpha_{ij}$ is symmetric, i.e. $\alpha_{ij} = \alpha_{ji}$.

we can always find 3-priciple axes, along which $\vec{P} // \vec{E}$, in other words. a coordinate in which $\alpha$ is diagonal $= \begin{pmatrix} \alpha_{xx}' & & 0 \\ & \alpha_{yy}' & \\ 0 & & \alpha_{zz}' \end{pmatrix}$.
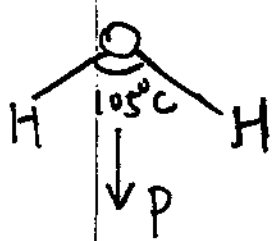
$(x', y', z')$

c.f. the moment of inertial, principle axes.

§ Permanet electric dipole moment     ↙ non-polar molecule

many molecules $CO_2$, $CH_4$ has no intrincic dipole moment. We need external electric field to induce a dipole ment.     But for HCl,

$H_2O$, their positive charge and negative charge centers do not coincide ⇒ polar - molecule

H—[105°C]—H  ↓P

$P \approx 6.1 \times 10^{-30}$ C.m

other polar molecules

HCl     $P_{HCl}/P_{H_2O} \approx 0.56$

$NH_3$     $P/P_{H_2O} \approx 0.78$

$CH_3OH$     $P/P_{H_2O} \approx 0.92$

✲ but actually even for a H atom, at any instant time, it has an dipole moment. But electrons moves so fast that for usual purpose, we observe an average effect so that dipole moment averges to zero.

Rigorously speaking, if we wait for long enough time, the dipole ④ moment of $H_2O$, $HCl$, etc should also averge to zero. However, this is the time scale related to the motion of atoms inside mole cule, $\underset{relative}{}$ which is much slower than the time-scale of electron motion. For the interaction between molecules, its chacteristic time scale is $\overset{\text{"FIXED"}}{}$ not long enough to allow to average out $\underset{all}{the}$ molecule configuration, thus we can think polar-molecules with $\overset{a}{}$ "permerant" dipole moment. But for atom, we do think that it's diapole averges to zero.

For polar molecule, under the external field $\vec{E}$, if $\vec{P} \not\parallel \vec{E}$, it feels a turge.

$$\vec{N} = \vec{r_+} \times \vec{F_+} + \vec{r_-} \times \vec{F_-} = q(\vec{r_+} - \vec{r_-}) \times \vec{E} = \vec{P} \times \vec{E}$$

total force $\vec{F} = \vec{F_+} - \vec{F_-} = q(\vec{E_+} - \vec{E_-}) = q \Delta\vec{E}$

$$= q\left(\hat{x}[\partial_x E_x \cdot d_x] + \hat{y}(\partial_y E_y \cdot d_y) + \hat{z}(\partial_z E_z \cdot d_z)\right)$$

$$= q(\vec{d}\cdot\vec{\nabla})\vec{E} = (\vec{P}\cdot\vec{\nabla})\vec{E}$$

§ Polarization:

$P = $ dipole moment / unit volume.