# Lect 1 Vecter algebra

Vecter algebra:

scalar : quantities have magnitude but no direction

mass, charge, density, ~~temperature~~, etc

non-relativistic

vecter : quantities have both magnitude & direction

$\vec{r}$, $\vec{v}$, $\vec{a}$. Vecter can be considered as an array of numbers

$\vec{r} = (x, y, z)$, $\vec{v} = (v_x, v_y, v_z) = v_x \vec{i} + v_y \vec{j} + v_z \vec{k}$

$= x\vec{i} + y\vec{j} + z\vec{k}$

unit basis

① $\vec{A} + \vec{B} = (A_x, A_y, A_z) + (B_x, B_y, B_z) = (A_x + B_x, A_y + B_y, A_z + B_z)$

$= \vec{B} + \vec{A}$

② $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = \vec{B} \cdot \vec{A} \Rightarrow \vec{A} \cdot \vec{A} = A_x^2 + A_y^2 + A_z^2$

norm $|A| = \sqrt{\vec{A} \cdot \vec{A}}$

③ $$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y,\ A_z B_x - B_z A_x,\ A_x B_y - A_y B_x)$$

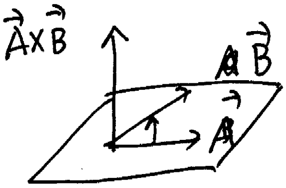
$= -\vec{B} \times \vec{A} \Rightarrow \vec{A} \times \vec{A} = 0$

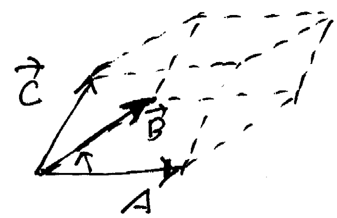
④ $$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$$

(5) $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

$(\vec{A}\times\vec{B})\times\vec{C} = -\vec{A}(\vec{B}\cdot\vec{C}) + \vec{B}(\vec{A}\cdot\vec{C})$

(6) $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D}) = (\vec{A}\cdot\vec{C})\cdot(\vec{B}\cdot\vec{D}) - (\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})$




$\vec{C}\cdot(\vec{A}\times\vec{B})$
volume of the parallelepiped

§ position, displacement;

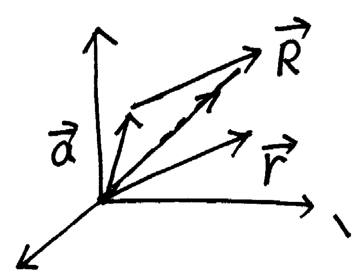
$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ $\quad r = \sqrt{x^2+y^2+z^2}$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\vec{i} + y\vec{j} + z\vec{k}}{\sqrt{x^2+y^2+z^2}}$$

$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$, $\quad |r_{12}| = |\vec{r} - \vec{r}'|$, $\quad \hat{r}_{12} = \dfrac{\vec{r}_1 - \vec{r}_2}{|r_1 - r_2|}$

§ transformations of vector

translation

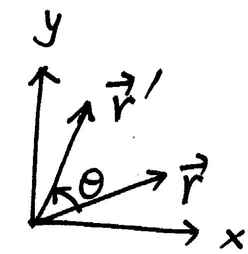



(1). vector $\vec{r}$ is translated by $\vec{a}$, the resultant

$\vec{R} = \vec{r} + \vec{a}$. using the matrix method

$$\begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} + \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$

2 rotation:




vecter $\vec{r}$ is rotated around the z-axis by an angle $\theta$: $\vec{r}\,' = R_z(\theta)\,\vec{r}$, its componets satisfy the matrix relation:

$$\begin{pmatrix} r_x' \\ r_y' \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ 0 \end{pmatrix}$$

generally, for a 3d rotation: $\vec{r}\,' = R\,\vec{r}$,

or
$$\begin{pmatrix} r_x' \\ r_y' \\ r_z' \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

↘ 3×3 orthogonal matrix: $R^T R = I$.
$\det R = 1$

3 inversion

$$\vec{r}\,' = -\vec{r}$$

or
$$\begin{pmatrix} r_x' \\ r_y' \\ r_z' \end{pmatrix} = \begin{pmatrix} -1 & & \\ & -1 & \\ & & -1 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

4 reflection with respect to xy, yz, zx-plans.

xy: $$\begin{pmatrix} r_x' \\ r_y' \\ r_z' \end{pmatrix} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix},$$

yz:
$$\begin{pmatrix} r_x' \\ r_y' \\ r_z' \end{pmatrix} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

zx
$$\begin{pmatrix} r_x' \\ r_y' \\ r_z' \end{pmatrix} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

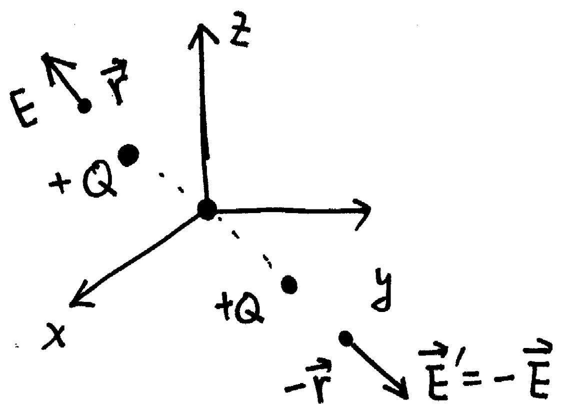
All the transformations which keep the norm of $\vec{r}$ unchanged, can be decomposed into a series of the above operations.

§ pseudo-vector:

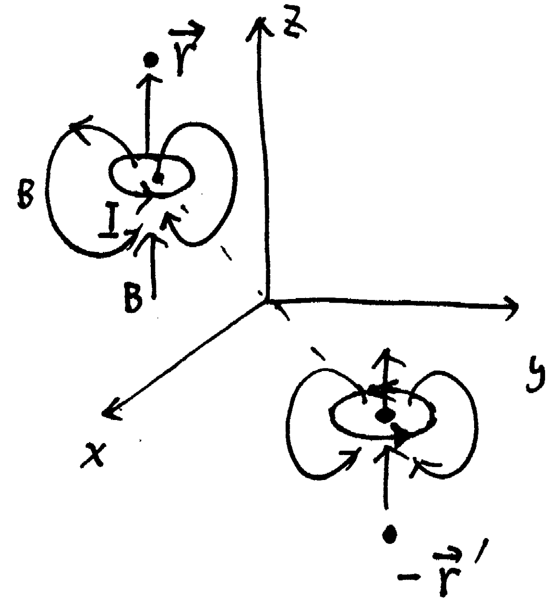
$\vec{E}$ - vector is an ordinary vector, satisfying the above transformation.

$\vec{B}$ - is pseudo vector. Under spatial rotations, $\vec{B}$ transforms as the same as $\vec{E}$.

But under inversion: $\vec{E} \to -\vec{E}$, $\vec{B} \to \vec{B}$.




inversion of a charge Q and its electric field




the inversion of a current loop and its surrounding B-field.

$\vec{B}$ is even under spatial inversion, thus is quite different from usuall vecters. — pseudo-vecter or axial vecter