Velocity Aberration of Lunar Return

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Because the Moon is a moving target, sliding past at 1 km s^{-1}, a slight velocity aberration is introduced to the return laser beam. Let us first look at this in the context of a specular reflection off of a flat mirror moving transverse to the laser transmit/receive station. Figure 1 shows the geometry for a normal incidence,

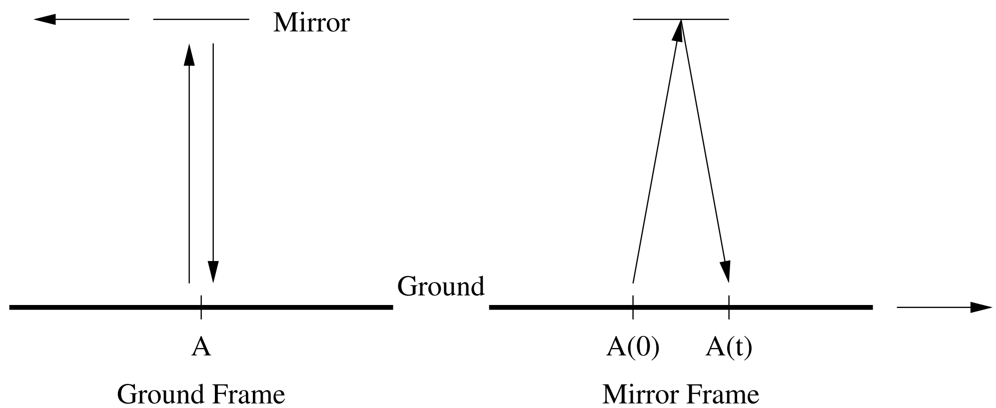

Figure 1: At left, in the ground frame, the normal incidence mirror reflection is unaffected by the mirror's motion. In the mirror frame, at right, the incoming beam has some transverse component, and the specular reflection sends the beam away with the same transverse component, such that the receiving station (at A) has moved to intercept the return beam.

transversely moving mirror in both the ground frame and the mirror frame. Both show the return beam arriving at the transmit station, though in the mirror frame the reflection is *not* normal incidence.

This is the important point when considering retro-reflector behavior. The retro-reflector has the property that an incoming ray is redirected in the exact opposite direction *in the rest frame of the retro-reflector*. Figure 2 demonstrates the effect this has on transmit/receive locations from the ground. In particular, the ray that strikes the retro-reflector at normal incidence in its rest frame has a transverse component in the ground frame. Although this ray is redirected on itself in the rest frame, it has the same transverse component (not opposite) in the ground frame, such that the return beam misses the transmit station.

The angular offset in radians is proportional to $\beta \equiv v/c$, so that the Moon's motion introduces a $1/300,000 = 3.3\,\mu$R, or 0.7 arcsec angular deviation. Because this deviation propagates the Earth-Moon distance twice, the effective angular offset is about 1.4 arcsec. At the Earth-Moon distance this translates to about 2.5 km. All is not lost because even if the retro-reflectors were diffraction limited 3.8 cm diameter apertures, their return-beam would be approximately 3.5 arcsec in width—larger than twice the velocity-induced offset. The lunar retro-reflectors do not exhibit diffraction-limited performance, with an estimated

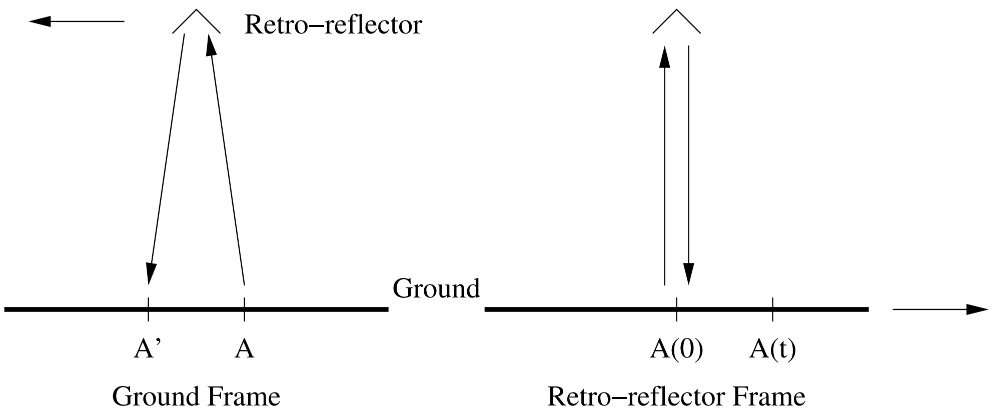

Figure 2: Only in the retro-reflector rest frame (right) is the light directed back on itself. A normal-incidence configuration in the retro-reflector frame requires a transverse ray component in the ground frame, resulting in separated transmit (A) and receive (A') locations, as indicated at left.

divergence of 10 arcsec. Assuming a Gaussian beam profile, this only introduces a 5% degradation in signal strength (compared to 35% degradation in the diffraction limited case).

It's not clear to me that this process can be viewed in terms of Lorentz contraction of the corner cubes. If you're going to work in the "ground" frame then there *is* a Lorentz contraction of the cubes in the transverse direction, altering the angles of the glass faces to order $\gamma - 1$, which is proportional to β^2. This becomes a 10^{-11} effect for the Moon, so-as to be considered negligible. Regardless of this effect, one still must consider the modified angle of incidence of the transmit beam, proportional to β. One way to see that the effect isn't totally characterized by Lorentz contraction is that the effect is symmetric. By closing the angle between reflective surfaces, an incident beam would split into two images—one trailing the motion and the other leading. This can be seen via a simple "mental raytrace" of two mirrors with an opening angle less than 90°. Depending on which mirror is hit first, the beam will emerge in one of two directions. The regime depicted in Figure 2 clearly indicates a one-sided offset, in contrast to the split image resulting merely from Lorentz contraction. I think these are separate effects (after all, the mirror in Figure 1 is free from Lorentz effects), and that the Lorentz component is of negligible contribution.

Despite the fact that there exists a 1.4 arcsec misalignment between the transmit beam and the return beam's *center*, the fact that the return beam *diverges* means that the part of the beam returned to the telescope appears to come from the *same* direction as the transmit beam went out. That's all well and good from a single shot standpoint. Without moving the telescope, one would shoot a pulse (in advance of the target) and see the return beam come from the same direction. But a problem arises when firing at repetition rates higher than 0.4 Hz. By the time the return pulse arrives back, the telescope is pointing in a new direction, so that the received beam appears to trail the transmit beam by 1.4 arcsec. This effect is diminished by the rotational motion of the Earth, amounting to 400 m s^{-1} at a latitude of 33°. Thus when the Moon is on the meridian, the 1.4 arcsec deviation is reduced to 0.9 arcsec. Figure 3 depicts the relevant geometry and intentional laser misalignment.

The matter of introducing an offset between incoming and outgoing beams is potentially very complicated, since the apparent direction of lunar motion depends on the Moon's direction in the sky (up to 56° of variation) as well as on the orientation of the alt-azimuth platform. Considering the possible angles of apparent motion on the sky, the direction of transverse motion (*not* simply angular motion) spans 172°,

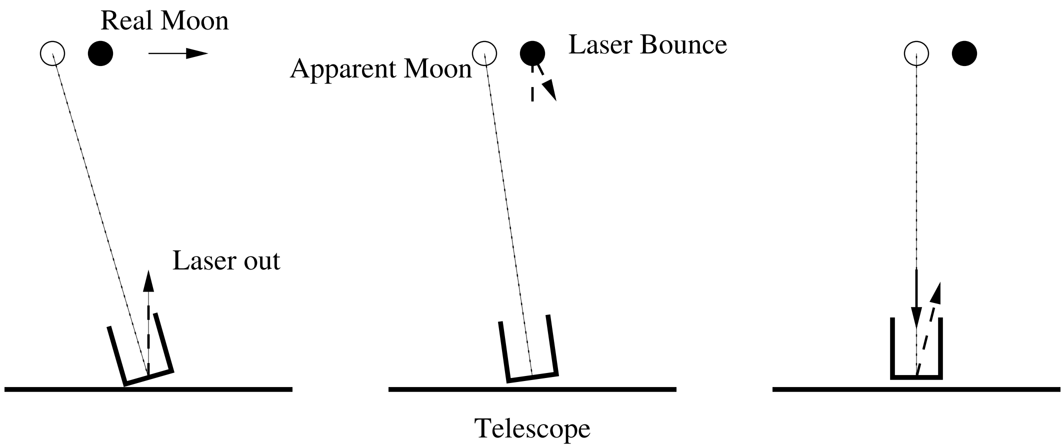

Figure 3: The telescope tracks the apparent Moon, which trails the real simultaneous Moon. The laser, however, is fired in advance of the real Moon so that the beam intercepts the real Moon at the appropriate time. The return pulse appears to originate from the apparent Moon.

symmetrically placed about an eastward motion. As noted above, the amplitude of this leading motion varies from 1.4 arcsec (on the horizon) to 0.85 arcsec (on the meridian). Implementing variable pointing of the laser is a very difficult task, and bound to cause problems. The *smart* way to do this is to co-boresight the laser with the telescope, so that its effective position is centered on the optical axis. The return beam will then be displaced from the optical axis, by an amount and direction described by the parameters above. This behavior determines the appropriate focal plane scale. Because the return beam is always displaced to *one side* of the optical axis (the Moon never reverses direction), the array can be offset in such a way that the optical axis is near one edge, and the return beam is roughly near the array center. Figure 4 shows the range of transmit/receive offsets that may be encountered (horizon to horizon) from APO with all possible apparent lunar positions and velocities.

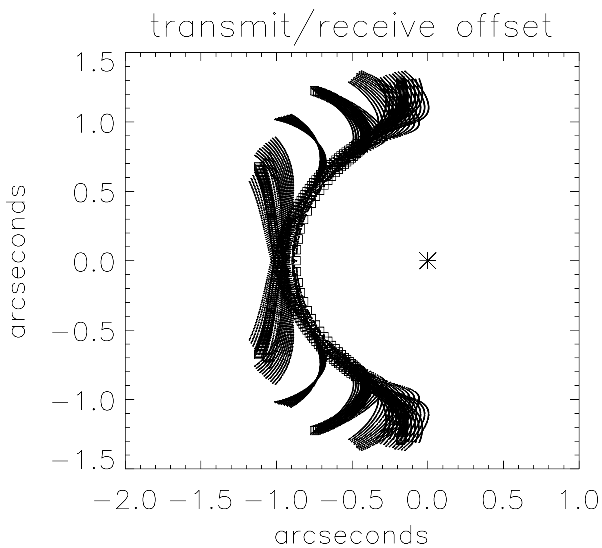

Figure 4: The possible return beam positions are shown here relative to the transmit beam direction. The horizontal axis is azimuth, and the vertical axis is elevation. The transmit position is marked by the asterisk. The boxes represent the beam displacements for the Moon at its maximum, minumum, and zero declination.