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See p. 27 for range calculation

See also p. 34 for errors

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CONTRIBUTIONS TO THE THEORY * OF ATMOSPHERIC REFRACTION

Part II. Refraction Corrections in Satellite Geodesy

A. Photogrammetric Refraction

Depending on its definition, satellite photogrammetry deals with vertical photography of the earth's surface taken from an orbiting satellite, or with photography of an orbiting satellite taken from the surface of the earth against the stellar background. The photogrammetric refraction is essentially the same in both cases.

To determine the geometric relationship between astronomical refraction Δz_1 and photogrammetric refraction $\Delta\theta_2$, we have from Fig. 3

$$\overline{OQ} = \frac{r_1 \sin(z_1 + \Delta z_1)}{\sin z_2} \qquad \overline{P_1 Q} = \frac{r_1 \sin(z_1 + \Delta z_1 - z_2)}{\sin z_2}$$

and

$$d = (r_2 - \overline{OQ}) \sin z_2 = r_2 \sin z_2 - r_1 \sin(z_1 + \Delta z_1)$$

$$s = \overline{P_1 Q} + d \operatorname{ctg} z_2 = r_2 \cos z_2 - r_1 \cos(z_1 + \Delta z_1)$$

The photogrammetric refraction is therefore obtained from the astronomical refraction by the formulas

$$\left\{ \begin{array}{ll} \Delta\theta_2 = \dfrac{d}{s} = \dfrac{r_2 \sin z_2 - r_1 \sin(z_1 + \Delta z_1)}{r_2 \cos z_2 - r_1 \cos(z_1 + \Delta z_1)} & (45) \\ r_2 \sin z_2 = n_1 r_1 \sin z_1 & (2') \end{array} \right.$$

applicable to both cases of satellite photogrammetry defined above.

* -- Suite et fin de l'article publié dans les n^{os} 105 et 106 du Bulletin Géodésique.

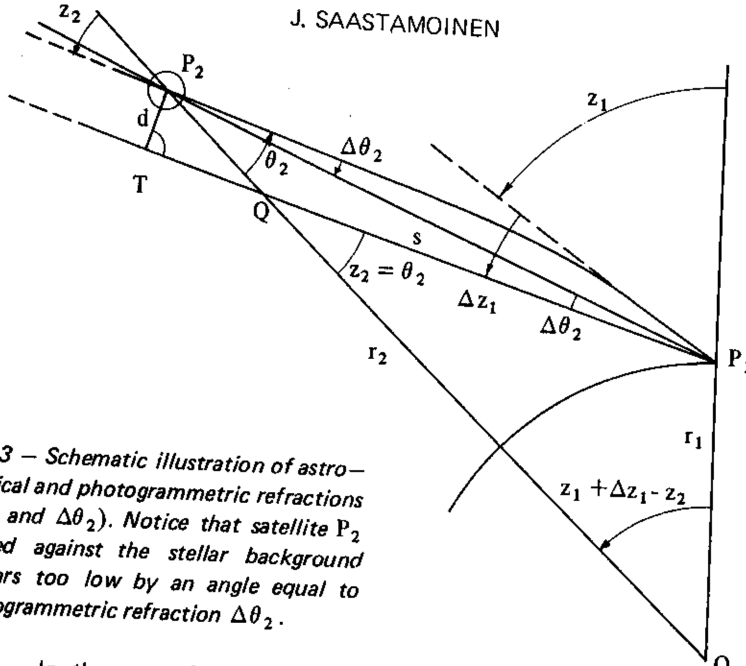

Fig. 3 – Schematic illustration of astronomical and photogrammetric refractions (Δz_1 and $\Delta\theta_2$). Notice that satellite P_2 viewed against the stellar background appears too low by an angle equal to photogrammetric refraction $\Delta\theta_2$.

In the case of satellite photography taken from a terrestrial camera station, distance s may be determined from orbital elements or computed bv the formula

$$s = \sqrt{r_2^2 - n_1^2 r_1^2 \sin^2 z_1} - \sqrt{r_1^2 - (n_1 r_1 \sin z_1 - d)^2} \simeq$$

$$\simeq r_1 \left(\sqrt{(r_2/r_1)^2 - \sin^2 z_1} - \cos z_1\right) \qquad (46)$$

For departure d , we have from (30)

$$\begin{cases} \sin(\Delta z_1) \simeq \Delta z_1 = \tan z_1 (n_1 - 1)\left[1 - \dfrac{RT_1}{r_1 g} \sec^2 z_1 + \dfrac{1}{2}\tan^2 z_1 (n_1 - 1)\right] + \delta \\ \cos(\Delta z_1) \simeq 1 - \dfrac{1}{2}(\Delta z_1)^2 \end{cases}$$

which gives

$$d = \left[\frac{(n_0 - 1) R T_0}{p_0 g} \frac{\tan z_1}{\cos z_1} p_1 - r_1 \cos z_1 \, \delta\right]\left[1 - \frac{(n_0 - 1) T_0}{p_0} \tan^2 z_1 \left(\frac{p_1}{T_1}\right)\right] \qquad (47)$$

where correction term $\delta = \delta_1 - \delta_2 - \delta_3 + \delta_4$ is expressed in radians. Numerically, assuming an effective wavelength of 0.554 microns,

$$d = 0.002317\left[\frac{\tan z}{\cos z}\left(\frac{p}{1000}\right) - 13.35 \cos z \, \delta''\right]\left[1 - 0.000079 \tan^2 z \left(\frac{p}{T}\right)\right] \qquad (47a)$$

where d is the departure in kilometres, p is the total barometric pressure in millibars, T is the absolute temperature in degrees Kelvin, z is the apparent zenith distance of the satellite, and δ'' is the correction term given in Table III on page 387 (B.G. n° 106). Table VI gives the resulting values of $\Delta\theta_2$ at standard atmospheric pressure and temperature for different zenith distances and orbital heights. The satellite will appear too low against the stars ; the image displacement (in microns) near the principal point of the photograph is equal to the tabular values multiplied by the focal length of the camera (in metres).

Table VI.

Differential Refraction in Microradians Between

Satellite and Stellar Background of White Stars

Apparent Zenith Distance	Orbital Height 250 km	500 km	750 km	1000 km	1500 km	2000 km
10°	1.7	0.8	0.6	0.4	0.3	0.2
20°	3.4	1.7	1.1	0.9	0.6	0.4
30°	5.5	2.7	1.8	1.4	0.9	0.7
40°	8.0	4.0	2.7	2.1	1.4	1.1
50°	11.5	5.9	4.0	3.0	2.1	1.6
60°	16.8	8.8	6.1	4.7	3.3	2.6
70°	28.4	15.4	11.0	8.7	6.3	5.0
80°	69.1	41.2	30.9	25.3	19.2	15.8

Sunlight reflected from the satellite is nearly white in colour, and the differential refractions in Table VI therefore apply to white stars only. Differences in colour as well as in zenith distance of individual stars can be taken into account by corrections to their astronomical refractions.

In the case of vertical photography of the earth's surface taken from an orbiting satellite, the photogrammetric refraction is obtained similarly by computing first zenith distance z_1 from nadir distance θ_2 measured from the photograph :

$$\sin z_1 = (r_2/r_1)\left[1 - \frac{(n_0 - 1) T_0}{p_0}\left(\frac{p_1}{T_1}\right)\right] \sin\theta_2 \qquad (48)$$

or numerically, for $\lambda = 0.554$ microns :

$$\sin z = (r_2/r_1)\,[1 - 0.000079\,(p/T)] \sin\theta \qquad (48a)$$

Table VII gives the resulting standard values of $\Delta\theta_2$ for super–wide angle photography from different camera heights. Due to these refraction angles, the photographic image points will appear to be shifted radially away from the principal point of the photograph ; the linear displacements (in microns) are equal to the tabular values multiplied by $f\,sec^2\,\theta$, f denoting the focal length of the camera (in metres).

Table VII.

Photogrammetric Refraction in Microradians

in Vertical Photography from Satellites

Apparent Nadir Distance	Orbital Height					
	250 km	500 km	750 km	1000 km	1500 km	2000 km
10°	1.7	0.9	0.6	0.5	0.3	0.3
20°	3.6	1.9	1.3	1.0	0.7	0.6
30°	5.8	3.1	2.2	1.7	1.3	1.1
40°	8.6	4.7	3.4	2.8	2.3	2.3
50°	12.7	7.3	5.8	5.4	6.9	-
59°	19.3	13.1	14.3	38.6	-	-

In photogrammetric mapping from low–altitude satellites, corrections for refraction will be required if horizontal control is extended by photogrammetric means. They can be obtained more conveniently using the approximate formulas

$$\left\{\begin{aligned} A^2 &= \left(\frac{r}{r+h}\right)^2 - sin^2\,\theta \\ \Delta\theta &= \frac{2.32\,p\,r\,sin\,\theta}{(r+h)^2\,A^2\,(cos\,\theta - A)} \end{aligned}\right. \tag{49}$$

where $\Delta\theta$ is the photogrammetric refraction in microradians at apparent nadir distance θ , p is the barometric pressure in millibars at the ground level, h is the height of the camera in kilometres above the ground level $(h > 50\,km)$, and r is the radius of the earth in kilometres.

B. — The Atmospheric Corrections for Troposphere and Stratosphere in Electro–magnetic Ranging of Satellites

Due to the retarding effect of the atmosphere on the propagation of electromagnetic waves, a range correction

$$\Delta S = \int_S (n-1)\,dS$$

where n is the refractive index referred to the group velocity of propagation, must be subtracted from observed electromagnetic distance S to obtain the true measured length of the effective ray path. As far as the electrically nonconducting lower atmosphere, the troposphere and the stratosphere, is concerned, this correction can be derived on the basis of the refraction theory developed previously in Part I for the determination of astronomical refraction.

Derivation of General Formula for Range Correction

In a spherically layered atmosphere, the basic mathematical expression of the range correction becomes

$$(0 \leq z_1 \leq 90^\circ)\,; \qquad \Delta S = \int_{r_1}^{r'} (n-1)\,sec\,z\,dr \tag{1*}$$

which corresponds to equation (1) of astronomical refraction. Setting $n\,r/(n_1\,r_1) = y$ for brevity, we now have from (2)

$$sin^2\,z = (sec^2\,z_1 - 1)/(y^2\,sec^2\,z_1)$$

$$cos^2\,z = (y^2\,sec^2\,z_1 - sec^2\,z_1 + 1)/(y^2\,sec^2\,z_1)$$

and

$$sec\,z = y\,sec\,z_1\,[1 + sec^2\,z_1\,(y^2-1)\,]^{-\frac{1}{2}} = y\,sec\,z_1 - \frac{1}{2}\,y\,(y^2-1)\,sec^3\,z_1 +$$

$$\frac{3}{8}\,y\,(y^2-1)^2\,sec^5\,z_1 - \frac{5}{16}\,y\,(y^2-1)^3\,sec^7\,z_1 + \frac{35}{128}\,y\,(y^2-1)^4\,sec^9\,z_1 - \ldots$$

Neglecting the subsequent terms in the binomial expansion, the first four may be written identically

$$\sec z = \sec z_1 + \left(\frac{r}{r_1} - y\right)(\sec^3 z_1 - \sec z_1) - \left(\frac{r - r_1}{r_1}\right)(\sec^3 z_1 - \sec z_1) -$$

$$- \left(1 + \frac{1}{2} y\right)(y-1)^2 \sec^3 z_1 + \frac{3}{8} y (y+1)^2 (y-1)^2 \sec^5 z_1 -$$

$$- \frac{5}{16} y (y+1)^3 (y-1)^3 \sec^7 z_1$$

into which we substitute the approximations

$$\frac{r}{r_1} - y = \left(\frac{r}{r_1}\right)\left(\frac{n_1 - n}{n_1}\right) = n_1 - n$$

$$\left(1 + \frac{1}{2} y\right)(y-1)^2 = \frac{3}{2}(y-1)^2 = \frac{3}{2 r_1^2}(r - r_1)^2$$

$$\frac{3}{8} y (y+1)^2 (y-1)^2 = \frac{3}{2}(y-1)^2 = \frac{3}{2}\left[\left(\frac{r}{r_1} - 1\right) - (n_1 - n)\right]^2 =$$

$$= \frac{3}{2 r_1^2}(r - r_1)^2 - \frac{3}{r_1}(n_1 - n)(r - r_1)$$

$$\frac{5}{16} y (y+1)^3 (y-1)^3 = \frac{5}{2 r_1^3}(r - r_1)^3$$

and obtain

$$\sec z = \sec z_1 - A_1 (r - r_1) + A_1' (n_1 - n) + A_2 (r - r_1)^2 -$$

$$- A_2' (n_1 - n)(r - r_1) - A_3 (r - r_1)^3 \qquad (3^*)$$

where the coefficients are :

$$A_1 = (\sec^3 z_1 - \sec z_1)/r_1$$

$$A_1' = \sec^3 z_1 - \sec z_1$$

$$A_2 = 3(\sec^5 z_1 - \sec^3 z_1)/(2 r_1^2) \qquad (4^*)$$

$$A_2' = 3 \sec^5 z_1 / r_1$$

$$A_3 = 5 \sec^7 z_1 / (2 r_1^3)$$

By the substitution of (3*), integral (1*) breaks down into a linear function of six atmospheric integrals :

$$\Delta S = \sec z_1 \int_{r_1}^{r'} (n-1)\,dr - A_1 \int_{r_1}^{r'} (n-1)(r - r_1)\,dr +$$

$$+ A_1' \int_{r_1}^{r'} (n-1)(n_1 - n)\,dr + A_2 \int_{r_1}^{r'} (n-1)(r - r_1)^2\,dr - \qquad (5^*)$$

$$- A_2' \int_{r_1}^{r'} (n-1)(n_1 - n)(r - r_1)\,dr - A_3 \int_{r_1}^{r'} (n-1)(r - r_1)^3\,dr$$

The first term in equation (5*) is given by refractivity integral (8) discussed previously. The five remaining integrals will now be determined, as follows.

<u>Integrals $\int_{r_1}^{r'} (n-1)(r - r_1)^q\,dr$.</u>

Using the substitutions

$$u = n - 1 \qquad v = (r - r_1)^{q+1}$$

$$du = dn \qquad dv = (q+1)(r - r_1)^q\,dr$$

and integrating by parts, we obtain for these integrals

$$(q = 1, 2, 3);\ \int_{r_1}^{r'} (n-1)(r - r_1)^q\,dr = \left(\frac{1}{q+1}\right) \int_1^{n_1} (r - r_1)^{q+1}\,dn \qquad (50)$$

The values of the integrals on the right side have been determined previously in Part I, equations (20), (28) and (29).

<u>**Integral** $\int_{r_1}^{r'} (n-1)(n_1-n)\,dr$.</u>

Since identically

$$(n-1)(n_1-n) = (n_1-1)(n-1) - (n-1)^2$$

we obtain immediately, applying integrals (9), (22) and (23)

$$\int_{r_1}^{r'} (n-1)(n_1-n)\,dr = \frac{R}{g}(n_1-1)^2 T_1 - \left(\frac{R}{2g+R\beta}\right)\Big[(n_1-1)^2 T_1 + \tfrac{1}{2}(R\beta/g)(n^0-1)^2 T^0\Big] \quad (51)$$

<u>**Integral** $\int_{r_1}^{r'} (n-1)(n_1-n)(r-r_1)\,dr$.</u>

Using the same identity as in the previous case, we have first

$$\int_{r_1}^{r'} (n-1)(n_1-n)(r-r_1)\,dr = (n_1-1)\int_{r_1}^{r'} (n-1)(r-r_1)\,dr - \int_{r_1}^{r'} (n-1)^2(r-r_1)\,dr$$

For the stratospheric component of the second integral above, we have from (11)

$$\int (n-1)^2 (r-r^0)\,dr = (n^0-1)^2 \int (r-r^0)\,e^{2m(r-r^0)}\,dr =$$

$$= (n^0-1)^2 \frac{e^{2m(r-r^0)}}{4m^2}\,[2m(r-r^0)-1] + C =$$

$$= (n-1)^2\left[\frac{1}{2m}(r-r^0) - \frac{1}{4m^2}\right] + C$$

and

$$\int_{r^0}^{r'} (n-1)^2 (r-r^0)\,dr = \frac{(n^0-1)^2}{4m^2} = \frac{R^2}{4g^2}(n^0-1)^2 T^{02}$$

Consequently, using the identity $r - r_1 = r^0 - r_1 + r - r^0$ and applying integral (22), the total stratospheric component is obtained as

$$\int_{r^0}^{r'} (n-1)^2 (r-r_1)\,dr = \frac{R^2}{4g^2}(n^0-1)^2 T^{02} + \frac{R}{2g}(r^0-r_1)(n^0-1)^2 T^0$$

For the tropospheric component of the same integral we have from (15) and (17)

$$\int (n-1)^2 (r-r_1)\,dr = \frac{(n_1-1)^2 T_1}{\beta^2}\int \left(\frac{T}{T_1}\right)^{2m'}\left(\frac{T}{T_1}-1\right) dT =$$

$$= \frac{(n_1-1)^2 T_1^2}{\beta^2}\left[\left(\frac{1}{2m'+2}\right)\left(\frac{T}{T_1}\right)^{2m'+2} - \left(\frac{1}{2m'+1}\right)\left(\frac{T}{T_1}\right)^{2m'+1}\right] + C =$$

$$= \frac{(n-1)^2 T_1 T}{\beta^2}\left[\left(\frac{1}{2m'+2}\right)\frac{T}{T_1} - \frac{1}{2m'+1}\right] + C =$$

$$= \frac{(n-1)^2 T_1 T}{\beta^2(2m'+1)}\left[\left(\frac{2m'+1}{2m'+2}\right)\frac{T}{T_1} - 1\right] + C = \frac{(n-1)^2 T_1 T}{\beta^2(2m'+1)}\left[\frac{T}{T_1} - 1 - \left(\frac{1}{2m'+2}\right)\frac{T}{T_1}\right] + C =$$

$$= \frac{(n-1)^2 (r-r_1) T}{\beta(2m'+1)} - \frac{(n-1)^2 T^2}{\beta^2(2m'+1)(2m'+2)} + C =$$

$$= -\left(\frac{R}{2g+R\beta}\right)(r-r_1)(n-1)^2 T - \frac{R^2}{2g(2g+R\beta)}(n-1)^2 T^2 + C$$

The tropospheric component is accordingly

$$\int_{r_1}^{r^0} (n-1)^2 (r-r_1)\,dr = \frac{R^2}{2g(2g+R\beta)}\Big[(n_1-1)^2 T_1^2 - (n^0-1)^2 T^{0^2}\Big] - \frac{R}{2g+R\beta}(r^0-r_1)(n^0-1)^2 T^0$$

which added to the stratospheric component gives the total value of the integral

$$\int_{r_1}^{r'} (n-1)^2 (r-r_1)\,dr = \frac{R^2}{2g^2}\left[\frac{(n_1-1)^2 T_1^{\,2} - (n^0-1)^2 T_1 T^0}{2+R\beta/g} + \tfrac{1}{2}(n^0-1)^2 T^{0^2}\right] \tag{52}$$

and finally, in view of equations (50) and (20)

$$\int_{r_1}^{r'} (n-1)(n_1-n)(r-r_1)\,dr = \frac{R^2}{g^2}(n_1-1)\left[\frac{(n_1-1)T_1^{\,2} - (n^0-1)T^{0^2}}{1-R\beta/g} + (n^0-1)T^{0^2}\right] - \frac{R^2}{2g^2}\left[\frac{(n_1-1)^2 T_1^{\,2} - (n^0-1)^2 T_1 T^0}{2+R\beta/g} + \tfrac{1}{2}(n^0-1)^2 T^{0^2}\right] \tag{53}$$

Combining the results from the preceding developments, we may now write on the basis of equation (5*) the following expression for the range correction :

$$\Delta S = \frac{R}{g}\sec z_1 (n_1-1)T_1 - \frac{R^2}{r_1 g^2}(\sec^3 z_1 - \sec z_1)\left[\frac{(n_1'-1)T_1'^{\,2} - (n^0-1)T^{0^2}}{1-R\beta/g} + (n^0-1)T^{0^2}\right] + \delta_5 + \delta_6 - \delta_7 - \delta_8 \tag{30*}$$

where

$$\delta_5 = \frac{R}{g}(\sec^3 z_1 - \sec z_1)\left[(n_1'-1)^2 T_1' - \frac{(n_1'-1)^2 T_1' + \frac{1}{2}(R\beta/g)(n^0-1)^2 T^0}{2+R\beta/g}\right]$$

$$\delta_6 = \frac{3R^3}{r_1^2 g^3}(\sec^5 z_1 - \sec^3 z_1)\left[\frac{(n_1'-1)T_1'^{\,3} - (n^0-1)T^{0^3}}{(1-R\beta/g)(1-2R\beta/g)} + (n^0-1)T^{0^3}\right] + \frac{3R^2}{r_1^2 g^2}(\sec^5 z_1 - \sec^3 z_1)\left(1 - \frac{1}{1-R\beta/g}\right)(r^0-r_1)(n^0-1)T^{0^2}$$

$$\delta_7 = \frac{3R^2}{r_1 g^2}\sec^5 z_1 (n_1'-1)\left[\frac{(n_1'-1)T_1'^{\,2} - (n^0-1)T^{0^2}}{1-R\beta/g} + (n^0-1)T^{0^2}\right] - \frac{3R^2}{2r_1 g^2}\sec^5 z_1\left[\frac{(n_1'-1)^2 T_1'^{\,2} - (n^0-1)^2 T_1' T^0}{2+R\beta/g} + \tfrac{1}{2}(n^0-1)^2 T^{0^2}\right]$$

$$\delta_8 = \frac{15R^4}{r_1^3 g^4}\sec^7 z_1\left[\frac{(n_1'-1)T_1'^{\,4} - (n^0-1)T^{0^4}}{(1-R\beta/g)(1-2R\beta/g)(1-3R\beta/g)} + (n^0-1)T^{0^4}\right] + \frac{15R^3}{r_1^3 g^3}\sec^7 z_1\left[1 - \frac{1}{(1-R\beta/g)(1-2R\beta/g)}\right](r^0-r_1)(n^0-1)T^{0^3} + \frac{15R^2}{2r_1^3 g^2}\sec^7 z_1\left(1 - \frac{1}{1-R\beta/g}\right)(r^0-r_1)^2(n^0-1)T^{0^2}$$

represent minor correction terms, and the primed quantities refer to values corrected for ground inversion. The formal accuracy of formula (30*) has been tested in Tables IXa – IXc on the three atmospheric models introduced earlier. The refractive indices were computed using the Essen and Froome formula for radio microwaves.

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Table IXa.

Atmospheric Model No. 1

(Tropical Zone)

Range Correction, $\Delta S = \int_{r_1}^{r'} (n-1) \sec z \, dr$, *for* $z = 60^\circ, 70^\circ$ & 80°

r, km 6360+	(n – 1)10⁶	sec z $z_1=60^\circ$	sec z $z_1=70^\circ$	sec z $z_1=80^\circ$	(n – 1) 10³ sec z $z_1=60^\circ$	(n – 1) 10³ sec z $z_1=70^\circ$	(n – 1) 10³ sec z $z_1=80^\circ$
0	261.46	2.00000	2.92380	5.75877	0.52293	0.76447	1.50572
0.8	242.49	1.99936	2.92145	5.73910	0.48482	0.70842	1.39167
1.6	224.61	1.99871	2.91908	5.71943	0.44893	0.65566	1.28465
2.4	207.78	1.99806	2.91669	5.69978	0.41516	0.60604	1.18431
3.2	191.96	1.99741	2.91429	5.68016	0.38342	0.55942	1.09035
4	177.09	1.99674	2.91187	5.66057	0.35361	0.51567	1.00245
4.8	163.15	1.99608	2.90944	5.64104	0.32565	0.47467	0.92032
5.6	150.08	1.99541	2.90700	5.62156	0.29946	0.43627	0.84366
6.4	137.84	1.99473	2.90454	5.60215	0.27495	0.40036	0.77220
7.7	119.64	1.99363	2.90053	5.57077	0.23851	0.34701	0.66647
9	103.37	1.99252	2.89650	5.53960	0.20597	0.29942	0.57265
10.3	88.90	1.99139	2.89246	5.50869	0.17703	0.25713	0.48970
11.6	76.06	1.99027	2.88839	5.47804	0.15137	0.21968	0.41664
12.9	64.72	1.98913	2.88431	5.44768	0.12873	0.18667	0.35257
14.2	54.75	1.98799	2.88023	5.41763	0.10884	0.15769	0.29662
15.5	46.03	1.98685	2.87613	5.38790	0.09145	0.13238	0.24799
16.8	38.43	1.98570	2.87204	5.35851	0.07631	0.11038	0.20594
16.8	38.43	1.98570	2.87204	5.35851	0.07631	0.11038	0.20594
17.7	32.92	1.98491	2.86922	5.33848	0.06534	0.09445	0.17573
18.6	28.19	1.98411	2.86639	5.31858	0.05594	0.08082	0.14996
19.5	24.15	1.98331	2.86356	5.29880	0.04790	0.06915	0.12796
20.4	20.68	1.98251	2.86073	5.27917	0.04101	0.05917	0.10920
21.3	17.72	1.98171	2.85790	5.25969	0.03511	0.05063	0.09319
22.2	15.18	1.98091	2.85507	5.24038	0.03006	0.04333	0.07952
23.1	13.00	1.98011	2.85224	5.22124	0.02574	0.03707	0.06786
24	11.13	1.97930	2.84941	5.20227	0.02204	0.03172	0.05792
26	7.89	1.97752	2.84314	5.16075	0.01560	0.02244	0.04072
28	5.59	1.97573	2.83691	5.12013	0.01105	0.01587	0.02864
30	3.96	1.97395	2.83070	5.08040	0.00783	0.01122	0.02014
32	2.81	1.97217	2.82454	5.04157	0.00554	0.00794	0.01417
34	1.99	1.97039	2.81841	5.00360	0.00392	0.00561	0.00997
36	1.41	1.96862	2.81232	4.96649	0.00278	0.00397	0.00701
38	1.00	1.96686	2.80628	4.93020	0.00197	0.00281	0.00493
40	0.71	1.96510	2.80027	4.89472	0.00139	0.00199	0.00347
44	0.36	1.96160	2.78839	4.82606	0.00070	0.00099	0.00172
48	0.18	1.95812	2.77668	4.76032	0.00035	0.00050	0.00085
52	0.09	1.95467	2.76514	4.69730	0.00018	0.00025	0.00042
56	0.05	1.95124	2.75375	4.63682	0.00009	0.00012	0.00021
60	0.02	1.94784	2.74253	4.57873	0.00004	0.00006	0.00010
64	0.01	1.94446	2.73147	4.52287	0.00002	0.00003	0.00005
68	0.006	1.94111	2.72055	4.46910	0.00001	0.00002	0.00003
72	0.003	1.93778	2.70979	4.41730	0.00001	0.00001	0.00001

Integrals * ;			
0 – 6.4 km :	2.48697	3.62931	7.08185
6.4 – 16.8 km :	1.65816	2.40806	4.58328
16.8 – 24 km :	0.31461	0.45419	0.84005
24 – 40 km :	0.11964	0.17171	0.30937
40 – 72 km :	0.00805	0.01143	0.01971
Range Correction, metres :	4.587	6.675	12.834
Formula (30*) ;			
1st term :	4.6020	6.7277	13.2511
2nd term :	–0.0164	–0.0603	– 0.5064
δ_5 :	0.0016	0.0060	0.0504
δ_6 :	0.0002	0.0016	0.0536
$-\delta_7$:	–0.0000	–0.0003	– 0.0093
$-\delta_8$:	–0.0000	–0.0001	– 0.0091
	4.587	6.675	–12.830

* See footnote to Table IIa.

Table IXb.

Atmospheric Model No. 2

(Temperate Zone)

Range Correction, $\Delta S = \int_{r_1}^{r'} (n-1) \sec z \, dr$, *for* $z = 60^\circ, 70^\circ$ & 80°

r, km 6380+	(n – 1) 10⁶	sec z $z_1=60^\circ$	sec z $z_1=70^\circ$	sec z $z_1=80^\circ$	(n – 1) 10³ sec z $z_1=60^\circ$	(n – 1) 10³ sec z $z_1=70^\circ$	(n – 1) 10³ sec z $z_1=80^\circ$
0	276.37	2.00000	2.92380	5.75877	0.55275	0.80806	1.59157
0.5	263.20	1.99961	2.92237	5.74674	0.52629	0.76916	1.51253
1	250.51	1.99922	2.92092	5.73469	0.50082	0.73172	1.43660
1.5	238.30	1.99882	2.91947	5.72264	0.47631	0.69570	1.36369
2	226.55	1.99842	2.91800	5.71058	0.45273	0.66106	1.29371
2.5	215.24	1.99802	2.91654	5.69851	0.43006	0.62777	1.22657
3	204.38	1.99762	2.91506	5.68645	0.40827	0.59578	1.16220
3.5	193.94	1.99721	2.91358	5.67439	0.38734	0.56506	1.10050
4	183.92	1.99680	2.91209	5.66234	0.36725	0.53558	1.04140
4.8	168.71	1.99615	2.90969	5.64308	0.33677	0.49089	0.95204
5.6	154.49	1.99549	2.90728	5.62384	0.30828	0.44914	0.86881
6.4	141.20	1.99482	2.90486	5.60466	0.28167	0.41077	0.79139
7.2	128.81	1.99415	2.90242	5.58552	0.25687	0.37387	0.71948
8	117.27	1.99347	2.89998	5.56644	0.23378	0.34009	0.65279
8.8	106.54	1.99280	2.89752	5.54743	0.21232	0.30871	0.59104
9.6	96.58	1.99211	2.89505	5.52849	0.19241	0.27962	0.53396
10.4	87.35	1.99143	2.89258	5.50964	0.17396	0.25268	0.48129
10.4	87.35	1.99143	2.89258	5.50964	0.17396	0.25268	0.48129
12.1	66.94	1.98998	2.88736	5.47035	0.13320	0.19327	0.36616
13.8	51.29	1.98851	2.88208	5.43119	0.10199	0.14782	0.27856
15.5	39.30	1.98702	2.87675	5.39233	0.07809	0.11306	0.21192
17.2	30.11	1.98552	2.87139	5.35390	0.05979	0.08647	0.16123
18.9	23.08	1.98401	2.86603	5.31601	0.04578	0.06613	0.12267
20.6	17.68	1.98250	2.86066	5.27870	0.03505	0.05058	0.09334
22.3	13.55	1.98098	2.85531	5.24202	0.02684	0.03869	0.07102
24	10.38	1.97946	2.84997	5.20599	0.02055	0.02959	0.05405
26	7.59	1.97768	2.84371	5.16446	0.01501	0.02158	0.03920
28	5.55	1.97589	2.83748	5.12384	0.01096	0.01574	0.02843
30	4.06	1.97411	2.83129	5.08413	0.00801	0.01149	0.02062
32	2.97	1.97234	2.82514	5.04532	0.00585	0.00838	0.01496
34	2.17	1.97057	2.81902	5.00738	0.00427	0.00611	0.01086
36	1.59	1.96880	2.81295	4.97030	0.00312	0.00446	0.00788
38	1.16	1.96704	2.80692	4.93404	0.00228	0.00325	0.00572
40	0.85	1.96529	2.80093	4.89858	0.00167	0.00237	0.00415
44	0.45	1.96180	2.78908	4.82998	0.00089	0.00126	0.00219
48	0.24	1.95833	2.77739	4.76427	0.00047	0.00067	0.00115
52	0.13	1.95489	2.76588	4.70129	0.00025	0.00036	0.00061
56	0.07	1.95148	2.75452	4.64084	0.00013	0.00019	0.00032
60	0.04	1.94808	2.74332	4.58277	0.00007	0.00010	0.00017
64	0.02	1.94471	2.73778	4.52693	0.00004	0.00005	0.00009
68	0.01	1.94136	2.72139	4.47318	0.00002	0.00003	0.00005
72	0.006	1.93804	2.71065	4.42139	0.00001	0.00002	0.00002

Integrals * ;			
0 – 4 km :	1.82061	2.65859	5.20519
4 – 10.4 km :	1.67324	2.43594	4.69389
10.4 – 24 km :	0.97682	1.41470	2.65685
24 – 40 km :	0.12024	0.17257	0.31087
40 – 72 km :	0.01053	0.01495	0.02578
Range Correction, metres :	4.601	6.697	12.893
Formula (30*) ;			
1st term :	4.6154	6.7472	13.2895
2nd term :	–0.0158	–0.0580	–0.4870
δ_5 :	0.0017	0.0064	0.0534
δ_6 :	0.0002	0.0016	0.0511
$-\delta_7$:	–0.0000	–0.0003	–0.0095
$-\delta_8$:	–0.0000	–0.0001	–0.0091
	4.601	6.697	12.888

* See footnote to Table IIa.

Table IXc.

Atmospheric Model No. 3

(Arctic Zone)

Range Correction, $\Delta S = \int_{r_1}^{r'} (n-1) \sec z \, dr$, *for* $z = 60^\circ, 70^\circ$ & 80°

r, km 6400+	$(n-1)10^6$	sec z $z_1=60^\circ$	sec z $z_1=70^\circ$	sec z $z_1=80^\circ$	$(n-1)10^3$ sec z $z_1=60^\circ$	$(n-1)10^3$ sec z $z_1=70^\circ$	$(n-1)10^3$ sec z $z_1=80^\circ$
0	313.57	2.00000	2.92380	5.75877	0.62714	0.91682	1.80578
0.2	302.60	1.99988	2.92336	5.75503	0.60517	0.88462	1.74149
0.4	292.11	1.99975	2.92290	5.75119	0.58415	0.85381	1.67998
0.6	282.06	1.99963	2.92243	5.74728	0.56402	0.82432	1.62111
0.8	272.44	1.99950	2.92195	5.74331	0.54475	0.79607	1.56474
1	263.23	1.99936	2.92147	5.73926	0.52629	0.76902	1.51074
1.2	254.40	1.99923	2.92098	5.73516	0.50860	0.74309	1.45901
1.4	245.93	1.99909	2.92048	5.73100	0.49164	0.71823	1.40943
1.6	237.81	1.99896	2.91997	5.72678	0.47537	0.69440	1.36189
1.6	237.81	1.99896	2.91997	5.72678	0.47537	0.69440	1.36189
2.5	216.62	1.99824	2.91735	5.70517	0.43287	0.63197	1.23588
3.4	196.91	1.99752	2.91470	5.68354	0.39333	0.57394	1.11915
4.3	178.60	1.99679	2.91204	5.66192	0.35663	0.52009	1.01122
5.2	161.62	1.99605	2.90935	5.64032	0.32260	0.47021	0.91158
6.1	145.90	1.99531	2.90664	5.61876	0.29112	0.42408	0.81978
7	131.38	1.99456	2.90392	5.59725	0.26204	0.38151	0.73536
7.9	117.99	1.99381	2.90118	5.57581	0.23525	0.34231	0.65788
8.8	105.67	1.99305	2.89843	5.55444	0.21060	0.30627	0.58692
8.8	105.67	1.99305	2.89843	5.55444	0.21060	0.30627	0.58692
10.7	78.95	1.99145	2.89266	5.51026	0.15722	0.22837	0.43503
12.6	58.99	1.98982	2.88679	5.46610	0.11737	0.17028	0.32243
14.5	44.07	1.98817	2.88086	5.42223	0.08762	0.12697	0.23897
16.4	32.93	1.98650	2.87488	5.37888	0.06541	0.09467	0.17712
18.3	24.60	1.98482	2.86889	5.33617	0.04883	0.07058	0.13129
20.2	18.38	1.98313	2.86290	5.29420	0.03645	0.05263	0.09732
22.1	13.73	1.98144	2.85692	5.25303	0.02721	0.03924	0.07215
24	10.26	1.97974	2.85096	4.21267	0.02032	0.02926	0.05349
26	7.55	1.97796	2.84471	5.17109	0.01493	0.02148	0.03904
28	5.56	1.97619	2.83850	5.13043	0.01098	0.01577	0.02850
30	4.09	1.97441	2.83232	5.09069	0.00807	0.01158	0.02081
32	3.01	1.97264	2.82618	5.05184	0.00593	0.00850	0.01519
34	2.21	1.97087	2.82007	5.01387	0.00436	0.00624	0.01109
36	1.63	1.96911	2.81401	4.97675	0.00321	0.00458	0.00810
38	1.20	1.96736	2.80799	4.94046	0.00236	0.00336	0.00592
40	0.88	1.96561	2.80202	4.90498	0.00173	0.00247	0.00432
44	0.48	1.96213	2.79019	4.83631	0.00094	0.00133	0.00231
48	0.26	1.95867	2.77852	4.77054	0.00051	0.00072	0.00123
52	0.14	1.95524	2.76703	4.70750	0.00027	0.00039	0.00066
56	0.08	1.95183	2.75569	4.64699	0.00015	0.00021	0.00035
60	0.04	1.94844	2.74451	4.58886	0.00008	0.00011	0.00019
64	0.02	1.94508	2.73349	4.53296	0.00004	0.00006	0.00010
68	0.01	1.94174	2.72261	4.47916	0.00002	0.00003	0.00005
72	0.007	1.93843	2.71189	4.42732	0.00001	0.00002	0.00003

Integrals * ;			
0 – 1.6 km :	0.87507	1.27879	2.51375
1.6 – 8.8 km :	2.37161	3.45773	6.71402
8.8 – 24 km :	1.23684	1.79310	3.38433
24 – 40 km :	0.12078	0.17337	0.31256
40 – 72 km :	0.01118	0.01588	0.02738
Range Correction, metres :	4.615	6.719	12.952
Formula (30*) ;			
1st term :	4.6287	6.7667	13.3277
2nd term :	–0.0155	–0.0568	–0.4770
δ_5 :	0.0017	0.0064	0.0537
δ_6 :	0.0002	0.0015	0.0501
$-\delta_7$:	–0.0000	–0.0003	–0.0095
$-\delta_8$:	–0.0000	–0.0001	–0.0088
	4.615	6.717	12.936

* See footnote to Table IIa.

Introduction to Practical Computation of Range Correction

Group Index of Refraction.

Radiant energy, such as an electromagnetic pulse or signal, travels with a speed referred to as the group velocity of propagation. The group velocity is different from the wave velocity in the case when the latter depends on wavelength λ, the group index of refraction being obtained from refractive index n by the formula

$$n - \lambda \frac{dn}{d\lambda}$$

The group index must be used in electromagnetic distance measurement in the computation of the velocity of propagation. It does not apply to the bending of a ray, which phenomenon depends on the wave velocity alone.

For visible light, $dn/d\lambda$ can be determined with sufficient accuracy by differentiating equation (33), which gives the following formula for the group refractivity of standard air :

$$n_g - 1 = \left(\frac{173.3 + 1/\lambda^2}{173.3 - 1/\lambda^2} \right) (n_0 - 1) \quad (54)$$

where wavelength λ is expressed in microns. In laser ranging, therefore, range correction (30*) must be multiplied by factor $(173.3 + 1/\lambda^2)/(173.3 - 1/\lambda^2)$ due to the effect of the group velocity of waves.

At the frequency range of radio microwaves, the atmosphere is considered nondispersive ; i.e. any waveform will be propagated without change of shape. In this case the group index is equal to the refractive index. Of the various formulas for the computation of the latter, we shall here quote the Essen and Froome formula adopted by the International Association of Geodesy [1963], converted into metric units :

$$(n-1)\,10^6 = 77.624\,(p/T) - 12.92\,(e/T) + 371900\,(e/T^2) \quad (55)$$

where p is the total pressure and e is the partial pressure of water vapour, both expressed in millibars, and T is the absolute temperature in degrees Kelvin.

Geometric Correction.

Since the effective ray path is curved due to the refraction, the difference in length between the measured arc and the corresponding chord

$$\delta_g = S - s$$

must further be subtracted from the measured distance. The total range correction then becomes

$$\Delta s = \Delta S + \delta_g \tag{56}$$

where δ_g is a geometric correction.

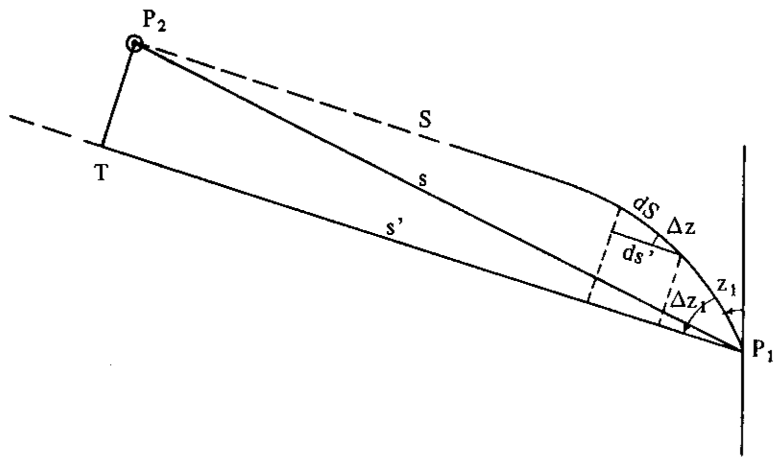

Fig. 4 – Geometric Correction $\delta_g = S - s \simeq S - s'$.

Referring to Figure 4, the geometric correction is approximately given by the integral

$$\delta_g = \int_S [1 - \cos(\Delta z)]\, dS = \frac{1}{2} \int_{r_1}^{r'} (\Delta z)^2 \sec z \, dr$$

where Δz is the astronomical refraction at points along the ray path. Now from (30)

$$\Delta z = \tan z (n-1) - \frac{R}{rg} \tan z \sec^2 z (n-1) T \tag{57}$$

$$\frac{1}{2}(\Delta z)^2 \sec z = \frac{1}{2} \tan^2 z \sec z (n-1)^2 - \frac{R}{rg} \tan^2 z \sec^3 z \,(n-1)^2\, T$$

and from (3) and (3*)

$$\tan^2 z \sec z = \tan^2 z_1 \sec z_1 - \frac{3}{r_1} \sec^5 z_1 (r - r_1)$$

$$\tan^2 z \sec^3 z = \sec^5 z_1$$

approximately, which gives

$$\delta_g = \frac{1}{2} \tan^2 z_1 \sec z_1 \int_{r_1}^{r'} (n-1)^2 \, dr - \frac{3}{2r_1} \sec^5 z_1 \int_{r_1}^{r'} (n-1)^2 (r - r_1)\, dr - \frac{R}{r_1 g} \sec^5 z_1 \int_{r_1}^{r'} (n-1)^2 T \, dr \tag{58}$$

The values of the first two integrals in equation (58) have been determined previously in equations (22), (23) and (52).

Equation (22) also gives the stratospheric component of the third integral in (58) :

$$\int_{r^0}^{r'} (n-1)^2 T\, dr = T^0 \int_{r^0}^{r'} (n-1)^2 \, dr = \frac{R}{2g} (n^0 - 1)^2 T^{02}$$

In the troposphere, we have from (17)

$$\int (n-1)^2 T \, dr = \frac{(n_1 - 1)^2 T_1}{\beta} \int \left(\frac{T}{T_1}\right)^{2m'+1} dT =$$

$$= \frac{(n_1 - 1)^2 T_1^2}{\beta(2m'+2)} \left(\frac{T}{T_1}\right)^{2m'+2} + C = -\frac{R}{2g} (n-1)^2 T^2 + C$$

and

$$\int_{r_1}^{r^0} (n-1)^2 T \, dr = \frac{R}{2g} [(n_1 - 1)^2 T_1^2 - (n^0 - 1)^2 T^{02}]$$

The total value of the third integral is consequently

$$\int_{r_1}^{r'} (n-1)^2 T\,dr = \frac{R}{2g}(n_1-1)^2 T_1^2 \tag{59}$$

Substituting the values of the atmospheric integrals into equation (58), we obtain the following formula for the geometric correction :

$$\delta_g = \frac{R}{2g}(\sec^3 z_1 - \sec z_1)\left[\frac{(n_1'-1)^2 T_1' + \frac{1}{2}(R\beta/g)(n^0-1)^2 T^0}{2+R\beta/g}\right] -$$

$$-\frac{3R^2}{4r_1 g^2}\sec^5 z_1\left[\frac{(n_1'-1)^2 T_1'^2 - (n^0-1)^2 T_1' T^0}{2+R\beta/g} + \frac{1}{2}(n^0-1)^2 T^{0^2}\right] -$$

$$-\frac{R^2}{2r_1 g^2}\sec^5 z_1 (n_1'-1)^2 T_1'^2 \tag{60}$$

where the primed quantities, as before, refer to values corrected for ground inversion. At a maximum zenith distance $z_1 = 80^\circ$, the geometric correction amounts to about 0.03 metres. It can be readily included in the range correction by modifying appropriate terms in equations (31*).

Corrections for Vapour Pressure.

The amount and distribution of water vapour in the atmosphere varies greatly according to the prevailing conditions of evaporation and condensation. Assuming that the vapour pressure decreases with height in a similar manner as total pressure (16), we can write

$$e = e_1\left(\frac{T}{T_1}\right)^{-\nu g/(R\beta)} \tag{61}$$

where ν is a numerical coefficient determined from local observations. Equation (61) provides a convenient means for the evaluation of various humidity integrals for the purpose of refraction.

To determine the contribution of humidity to refractivity integral (8), we have from (61)

$$\int\left(\frac{e}{T}\right)dr = \frac{e_1}{\beta T_1}\int\left(\frac{T}{T_1}\right)^{-\nu g/(R\beta)-1} dT = -\frac{R}{\nu g}\,e + C$$

$$\int_{r_1}^{r'}\left(\frac{e}{T}\right)dr = \frac{R}{\nu g}\,e_1 \tag{62}$$

and

$$\int\left(\frac{e}{T^2}\right)dr = \frac{e_1}{\beta T_1^2}\int\left(\frac{T}{T_1}\right)^{-\nu g/(R\beta)-2} dT =$$

$$= -\frac{e_1}{\beta T_1}\left[\frac{1}{\nu g/(R\beta)+1}\left(\frac{T}{T_1}\right)^{-\nu g/(R\beta)-1}\right] + C = -\left(\frac{R}{\nu g + R\beta}\right)\left(\frac{e}{T}\right) + C$$

$$\int_{r_1}^{r'}\left(\frac{e}{T^2}\right)dr = \left(\frac{R}{\nu g + R\beta}\right)\left(\frac{e_1}{T_1}\right) \tag{63}$$

This gives a correction for vapour pressure

$$\delta_w = \left[\frac{(n_0-1)RT_0}{\nu p_0 g}\left(1-\frac{R}{R_w}\right) - \frac{Rc_w}{\nu g} + \left(\frac{c_w'}{\nu g/R+\beta}\right)\frac{1}{T_1}\right]\sec z_1\, e_1 \tag{64}$$

which must be added to range correction (5*).

The contribution of humidity to the second term in (5*)

$$-\frac{Rc_w'}{\nu r_1 g(\nu g/R+\beta)}\tan^2 z_1 \sec z_1\, e_1$$

and to geometric correction (58)

$$+\frac{Rc_w'(n_1-1)}{g(\nu+1+2R\beta/g)T_1}\tan^2 z_1 \sec z_1\, e_1$$

are further corrections for vapour pressure, both derived on the basis of equations (61) and (7), that might be considered in the range correction. They amount to less than one percent of correction (64) and, in view of the uncertainty in the determination of the latter, may be omitted in practical applications.

At a maximum zenith distance $z_1 = 80^\circ$, and assuming an average value of $\nu = 4$, vapour pressure correction (64) may under extreme conditions exceed 2.5 metres in the radio range, and 0.03 metres in the laser range.

Formulas and Tables for the Computation of Range Correction

1. Standard Formulas.

The atmospheric correction for troposphere and stratosphere in electromagnetic ranging of satellites is given by the standard formula (a) for laser ranging :

$$\Delta s_0 = 0.002357 \sec z\,(p + 0.06\,e - B \tan^2 z) + \delta_L \quad (56a)$$

or (b) for radio ranging :

$$\Delta s_0 = 0.002277 \sec z \left[p + \left(\frac{1255}{T} + 0.05 \right) e - B \tan^2 z \right] + \delta_R \quad (56b)$$

where Δs_0 is the range correction in metres, z is the apparent (radio) zenith distance of the satellite, p is the total barometric pressure in millibars, e is the partial pressure of water vapour in millibars, T is the absolute temperature in degrees Kelvin, and B and δ are correction quantities obtained from Tables X and XI, respectively. In radio ranging, apparent zenith distance z can be determined from true zenith distance Z of the satellite by the formula $z = Z - \Delta z$, where

$$\Delta z'' = \frac{16''.0 \tan Z}{T} \left(p + \frac{4800\,e}{T} \right) - 0''.07\,(\tan^3 Z + \tan Z) \left(\frac{p}{1000} \right) \quad (57a)$$

is the angle of refraction.

2. Correction for the Effective Wavelength.

Formula (56a) employs a standard wavelength of 0.6943 microns for a ruby laser. For other laser systems, the numerical coefficient of the first term in the formula is obtained from the expression

$$\frac{0.39406\,(173.3 + 1/\lambda^2)}{(173.3 - 1/\lambda^2)^2}$$

where λ is the effective wavelength of the system expressed in microns.

3. Correction for Local Latitude and Station Height.

The numerical coefficient of the first term in formulas (56a) and (56b) is to some extent dependent on local latitude and station height. A locally corrected value may be obtained by applying a correction factor

$$1 + 0.0026 \cos 2\varphi + 0.00028\,H$$

Table X.

Standard Values of Correction Factor B

$$B = \frac{R}{r_1 g} \left[\frac{p_1' T_1' - (R\beta/g)\, p^0\, T_0}{1 - R\beta/g} \right]$$

Station Height Above Sea Level	B, mb	Station Height Above Sea Level	B, mb
0 km	1.156	2 km	0.874
0.5 km	1.079	2.5 km	0.813
1 km	1.006	3 km	0.757
1.5 km	0.938	4 km	0.654
2 km	0.874	5 km	0.563

Table XI.

Correction Term δ_L in Metres

$(\delta_R \simeq \delta_L)$

Apparent Zenith Distance	Station Height Above Sea Level							
	0 km	0.5 km	1 km	1.5 km	2 km	3 km	4 km	5 km
60°00'	+0.003	+0.003	+0.002	+0.002	+0.002	+0.002	+0.001	+0.001
66°00'	+0.006	+0.006	+0.005	+0.005	+0.004	+0.003	+0.003	+0.002
70°00'	+0.012	+0.011	+0.010	+0.009	+0.008	+0.006	+0.005	+0.004
73°00'	+0.020	+0.018	+0.017	+0.015	+0.013	+0.011	+0.009	+0.007
75°00'	+0.031	+0.028	+0.025	+0.023	+0.021	+0.017	+0.014	+0.011
76°00'	+0.039	+0.035	+0.032	+0.029	+0.026	+0.021	+0.017	+0.014
77°00'	+0.050	+0.045	+0.041	+0.037	+0.033	+0.027	+0.022	+0.018
78°00'	+0.065	+0.059	+0.054	+0.049	+0.044	+0.036	+0.030	+0.024
78°30'	+0.075	+0.068	+0.062	+0.056	+0.051	+0.042	+0.034	+0.028
79°00'	+0.087	+0.079	+0.072	+0.065	+0.059	+0.049	+0.040	+0.033
79°30'	+0.102	+0.093	+0.085	+0.077	+0.070	+0.058	+0.047	+0.039
79°45'	+0.111	+0.101	+0.092	+0.083	+0.076	+0.063	+0.052	+0.043
80°00'	+0.121	+0.110	+0.100	+0.091	+0.083	+0.068	+0.056	+0.047

where φ is the latitude, and H is the station height in kilometres above the sea level. The correction factor is equal to unity for sea–level stations in the middle latitudes.

Accuracy of the Determination of Range Correction

The bending of a ray, as observed in the astronomical refraction, is caused by the same physical phenomenon which alters the speed of propagation. The errors which affect the determination of the range correction are therefore closely related to those involved in the determination of astronomical refraction.

In laser ranging, the maximum errors in range correction at $z_1 = 80°$ may be considered to consist of the following : 10° el.

Departure from hydrostatic equilibrium	1.5 cm	*
Tilt of isopycnic surfaces	2.0 cm	*
Error in formula (56a)	1.0 cm	
Local error in correction factor B	2.0 cm	
Local error in correction term δ_L	0.5 cm	
Total maximum error	3.4 cm	

→ 6mm straight u

From these round figures it can be roughly estimated that for zenith distances not exceeding 80 degrees, the standard error of range correction (56a) will amount to about 1 to 2 centimetres.

The standard error of range correction (56b) for radio ranging is probably about ten times greater. In this case, the most important single source of error is due to local variations in the vertical distribution of humidity which may cause an error of up to one–fifth of the standard correction for vapour pressure.

Reference in Part II

International Association of Geodesy, Resolution No. 1 of the 13th General Assembly, Bulletin Géodésique, No. 70, p. 390, 1963.

* see No. 106 p 390 for discussion of these

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TRANSMISSION OF PLUMB–LINE DEFLECTIONS BY GROUND OBSERVATIONS IN POLARIZED LIGHT

Abstract

The problem of transferring astrogeodetic plumb–line deflections along the net by measuring angle components between the plumb–line, produced by the direction of electric vector of a linearly polarized light wave is discussed. A mathematical solution of the problem is given and a block–diagram of a photo–electric device for measuring angle components between plumb–lines is considered.

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In establishing geodetic nets in mountainous regions the problems involved in the determination of plumb–line deflection at every station of the net take on an especially great importance. The difficulties encountered in obtaining astronomic coordinates at every point by purely astronomic means and, as is often the case, in the absence of the necessary gravimetric data, call for some other ways to be investigated for the solution of the problem [1].

In what follows we describe one of practicable methods to transfer astronomic coordinates or astro–geodetic plumb–line deflections along a geodetic net, point to point, where projections of the angle between plumb–lines into a plane perpendicular to the line of sight are used as measured quantities.

Since under general conditions the atmosphere is not optically active, and reflection angles are small, the polarization plane of plane–polarized light does not show any appreciable bending.

As a result, the above mentioned projected angles can be measured as follows. Let a source of plane–polarized light be set at one of the stations. The plane in which the electric vector oscillations take place must be oriented parallel to the plumb–line at that station, and the beam pointed at station 2. If the axis of the receiving system analyser at station 2 is oriented parallel to the plumb–lines at that point, then the value of the light–beam passing through the analyser will be the measure of the angle between the plumb–lines projected into the plane which is normal to the line joining the two stations.

To obtain the necessary formulae let us denote the astronomic and the geodetic coordinates of 3 stations by φ_i, λ_i and B_i and L_i, respectively, where