

## Testing the equivalence principle: why and how?

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**Abstract.** Part of the theoretical motivation for improving the present level of testing of the equivalence principle is reviewed. The general rationale for optimizing the choice of pairs of materials to be tested is presented. A simplified rationale is introduced based on a trichotomy of competing classes of theoretical models.

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### 1. Why test the equivalence principle?

Einstein introduced in 1907 [1] what he called the ‘hypothesis of complete physical equivalence’ between a gravitational field and an accelerated system of reference. He used this ‘equivalence hypothesis’ [1, 2] as a heuristic tool to construct a physically satisfactory relativistic theory of gravitation. The resulting theory, general relativity, has been very successful, both in revising completely our description of the Universe, and in passing with flying colours all the experimental tests it has been submitted to. For instance, the universality of free fall (the experimental basis for the equivalence principle) has been verified at the  $10^{-12}$  level [3, 4], the quasi-static weak-field (‘post-Newtonian’) predictions have been checked in solar-system experiments at the  $10^{-3}$  level [5], the radiative structure (propagation of gravity with the speed of light as a helicity-2 interaction) has been verified by binary-pulsar data at the  $10^{-3}$  level [6], and the quasi-static strong-field predictions have been checked in binary-pulsar experiments at the  $10^{-2}$  level [7].

In view of this impressive record, should one apply Occam’s razor and decide that Einstein’s theory must be 100% right, and then stop testing it any further? My answer is definitely no! Indeed, one should continue testing a basic physical theory such as general relativity to the utmost precision available, simply because it is one of the essential pillars of the framework of physics.

A less extreme attitude than Occam’s razor might then be to focus on the experimental tests which presently have the lowest accuracy. In other words, this attitude would say that, because  $10^{-12} \ll 10^{-3}$ , one should decide that Einstein’s equivalence principle is 100% right, and concentrate on the other tests of relativistic gravity. This point of view led to the development of the parametrized post-Newtonian (PPN) formalism by Nordtvedt, Thorne, Will and others, building on foundations laid by Schiff and Dicke (see e.g. [8]). I want, however, to emphasize that any bias towards preferentially testing the class of so-called ‘metric theories of gravitation’, i.e. theories respecting the equivalence principle, is quite unjustified, both from a historical perspective, and (which is most important) from the point of view of the current overall framework of fundamental physics.

First, from a historical perspective, the introduction of well motivated alternatives to general relativity is due to Kaluza [9] and Jordan [10]. The motivation was that a scalar field with gravitational-strength couplings appears naturally as a new degree of freedom when one tries to unify gravity with electromagnetism. A scalar field of the same type (the ‘dilaton’) also appeared naturally when Scherk and Schwarz [11] proposed the idea that string theory should apply at the Planck scale. Still later, Scherk [12], working within the framework of extended supergravities, introduced the possibility of a vector field with gravitational-strength couplings. From the point of view of the present framework of physics, there are many reasons to expect the existence of new interactions with strength related to the gravitational one. In particular, extra vector fields appear naturally in supersymmetry-inspired extensions of the standard model [13, 14], and a plethora of (*a priori*) massless scalar fields show up in string theories. Now, the main point I wish to emphasize is that all the new interactions that naturally appear in extensions of the present framework of physics violate the equivalence principle. I know of no cases where an exact ‘metric’ coupling appeared naturally. The historical reason why so much emphasis has been put in the literature on artificially defined ‘metric’ theories of gravity comes from an important work of Fierz [15] on Jordan’s theory.

In this work, Fierz pointed out that Jordan’s original theory (with a Kaluza–Klein type scalar field) violated the equivalence principle in an observationally unacceptable way. He then introduced, in an *ad hoc* manner, the general class of metrically-coupled tensor–scalar gravity theories (with one arbitrary function) and the special one-parameter subclass of Jordan–Fierz theories (often named after Brans and Dicke [16]).

The conclusion of all this is that the experiments which are the most sensitive probes of new physics beyond the present framework are tests of the equivalence principle. The fact that present tests are at the  $10^{-12}$  level does not diminish the plausibility of small violations of the equivalence principle because there exist string-inspired models [17] in which one finds, in a non-fine-tuned way, violations of the universality of free fall at the level

$$\frac{\Delta a}{a} \sim 10^{-18} \kappa^{-4} (\Delta\varphi)^2, \quad (1)$$

where  $\kappa$  and  $\Delta\varphi$  are dimensionless quantities which could be of the order of unity.

To illustrate the superior probing power of equivalence principle tests let us mention that, barring the contrived possibility of a ‘metric coupling’, there is always proportionality between universality-of-free-fall deviations  $\Delta a/a$  and post-Newtonian deviations from general relativity (measured, say, by the Eddington parameter  $\bar{\gamma} \equiv \gamma_{\text{Eddington}} - 1$ ). For instance, in the general class of string-inspired models one can write [17, 18]

$$\left(\frac{\Delta a}{a}\right)_{AB} = \hat{\delta}_A - \hat{\delta}_B, \quad (2)$$

with

$$\hat{\delta}_A = -\bar{\gamma} \left[ c_B \left(\frac{B}{\mu}\right)_A + c_D \left(\frac{D}{\mu}\right)_A + 0.943 \times 10^{-5} \left(\frac{E}{\mu}\right)_A \right]. \quad (3)$$

Here, the suffixes  $A, B$  label two material bodies whose free falls are compared, while (in equation (3)),  $\mu$  denotes the mass in atomic mass units,  $B \equiv N + Z$  the baryon number,  $D = N - Z$  the neutron excess and  $E = Z(Z - 1)/(N + Z)^{1/3}$  a quantity proportional to nuclear electrostatic energy. The third term on the right-hand side of equation (3) is expected to dominate the other two. As the changes in  $E/\mu$  can be  $\gtrsim 1$  (see below) we see that, roughly speaking,  $\Delta a/a \sim 10^{-5} \bar{\gamma}$  in dilaton-like scalar models. In vector models [14], one has (from equation (3.13) of [19] linking  $\bar{\gamma}$  to the coupling of a new interaction)

$\Delta a/a \sim 10^{-2}\bar{\gamma}$  in the generic case of a coupling significantly involving the lepton number  $L = Z$ , and  $\Delta a/a \sim 10^{-3}\bar{\gamma}$  in the particular case of a coupling only to  $B = N + Z$ . Therefore equivalence principle tests constrain  $\bar{\gamma}$  to the  $10^{-7}$  level in scalar models (see [18] for precise numbers), and to the  $\lesssim 10^{-9}$  level in vector models. The fact that this is much smaller than the  $|\bar{\gamma}| \lesssim 10^{-3}$  level derived from post-Newtonian or pulsar tests gives a measure of the superior probing power of equivalence principle tests.

## 2. How to test the equivalence principle?

Taking the optimistic view that improved equivalence principle tests, henceforth abbreviated as EP tests, and notably STEP, will give positive (i.e. non-null) results, it is important to choose the pairs of materials tested so as to maximize, at once: (i) our confidence in the reality of the EP violation signals, (ii) the quantity of theoretical information that we can extract from the experimental data. These questions have been addressed in [20] in some detail. Here we want to summarize the main points of [20] and to propose a new, simplified approach appropriate to possible descoped versions of STEP.

We assume that we are looking for EP violation signals caused by some new long-range interaction. The interaction energy between some laboratory body  $A$  and an external body  $E$  (the Earth in STEP) reads as

$$V_{AE} = -\frac{Gm_A m_E}{r_{AE}} - \frac{H Q_A Q_E}{r_{AE}} \equiv -G_{AE} \frac{m_A m_E}{r_{AE}}. \quad (4)$$

Here we have introduced an effective (composition-dependent) gravitational constant for the  $(AE)$  pair:

$$G_{AE} = G + H \frac{Q_A Q_E}{m_A m_E} = G \left[ 1 + \frac{H}{Gu^2} \frac{Q_A Q_E}{\mu_A \mu_E} \right], \quad (5)$$

where  $G$  is Newton's bare gravitational constant,  $H$  is the new coupling constant ( $H > 0$  for scalar exchange and  $H < 0$  for vector exchange) and  $u$  denotes one atomic mass unit so that  $\mu_A \equiv m_A/u$ . In the equations above  $Q_A$  denotes the total 'charge' of body  $A$  to which the new interaction is coupled.

The fractional difference in free-fall acceleration of the pair  $(AB)$ ,  $(\Delta a/a)_{AB} \equiv 2(a_A - a_B)/(a_A + a_B)$  is given by

$$\left( \frac{\Delta a}{a} \right)_{AB} \simeq \frac{G_{AE} - G_{BE}}{G} = \hat{\delta}_A - \hat{\delta}_B, \quad (6)$$

with

$$\hat{\delta}_A \equiv \frac{H}{Gu^2} \frac{Q_E Q_A}{\mu_E \mu_A} \equiv h \hat{Q}_E \hat{Q}_A. \quad (7)$$

Here we have defined the shorthand  $h \equiv H/(Gu^2)$  and  $\hat{Q}_A \equiv Q_A/\mu_A$ .

The main issue of concern here is: how to optimize the choice of materials to be tested (see also [21])? As emphasized in [20], if one does not assume any theoretical model for the material dependence of the specific charge  $\hat{Q}_A = Q_A/\mu_A$ , the optimum strategy is (i) to restrict oneself to *connected* configurations of test materials, i.e. a set of pairs  $\mathcal{C} = \{(A_i A_j)\}$  such that any two elements  $A_k, A_l$  can be connected by a sequence of pairs belonging to  $\mathcal{C}$ , and (ii) to include *topological loops* in the configuration  $\mathcal{C}$  (e.g. a null pair  $(AA)$ , a double pair  $\{(AB), (AB)\}$ , a triangular loop  $\{(AB), (BC), (CA)\}$ , etc). The reason for conclusion (i) is that the measurements of the left-hand sides of (6) determine only *differences* between  $\hat{\delta}_A$ 's, so that the choice of disconnected configurations

(e.g.  $\{(AB), (BC), (DE)\}$ ) introduces more than one arbitrary additional constant in the phenomenological determination of the  $\hat{\delta}_A$ 's. The reason behind conclusion (ii) is that it allows one to exhibit convincing checks of the reality of a violation of the EP which are independent of any theory about the material dependence of  $Q_A$ : e.g. the simple redundancy check  $(\Delta a/a)_{(AB)_1} = (\Delta a/a)_{(AB)_2}$  when using two (different in some respect) pairs  $AB$ , or a richer cyclic check  $(\Delta a/a)_{AB} + (\Delta a/a)_{BC} + (\Delta a/a)_{CA} = 0$ . In view of the practical difficulties in realizing null pairs or cyclic configurations (see [21–23]), the inclusion of binary loops (repeated  $(AB)$  with some difference) appears as the simplest way of confirming the reality of an EP violation in a theory-independent manner.

It should be noted that the model-independent approach just sketched cannot (even if one includes in  $\mathcal{C}$  the maximum possible number of independent pairs, say 91 to cover the periodic table) give access to the basic theoretical quantities  $H$  and  $Q_A$  (or rather  $HQ_E^2$  and  $Q_A/Q_E$ , when taking into account the possibility of arbitrary rescalings  $Q_A \rightarrow \lambda Q_A$ ,  $H \rightarrow \lambda^{-2}H$ ). At best, one can determine the  $\hat{\delta}_A$ 's modulo an arbitrary (common) additive constant. This is not even enough information to determine the sign of  $H$ , i.e. the spin of the mediating field.

Let us now shift to a model-dependent approach, i.e. assume some theoretical model predicting a composition dependence of the specific† charge  $\hat{Q}_A \equiv Q_A/\mu_A$  of the form

$$\hat{Q}_A = \beta_0 + \sum_{i=1}^n \beta_i \xi_A^i. \quad (8)$$

Here  $\beta_0, \beta_i$  are some coupling parameters and  $\xi_A^i$  some specific elementary charges. For instance, in vector models (i.e. models where the apparent EP violation is due to the exchange of an extra  $U(1)$  long-range gauge field), we expect to have only two independent elementary charges [14] (for neutral bodies), baryon number and lepton number, and no composition-independent coupling to mass: i.e. we expect  $\beta_0 = 0$  and  $n = 2$  with, say,  $\xi^1 = (N + Z)/\mu$  and  $\xi^2 = (N - Z)/\mu$ . (Here, as above,  $N =$  neutron number,  $Z =$  proton number = atomic number = lepton number.) On the other hand, in the case of an EP violation due to any of the long-range gauge-neutral scalar fields of string theory (moduli), we expect to have a universal piece in  $Q_A$ , i.e.  $\beta_0 \neq 0$ , and three independent elementary charges [17]. This yields for equation (8):  $n = 3$  and

$$\xi^1 = (N + Z)/\mu, \quad (9a)$$

$$\xi^2 = (N - Z)/\mu, \quad (9b)$$

$$\xi^3 = E/\mu \simeq Z(Z - 1)/((N + Z)^{1/3}\mu). \quad (9c)$$

In equation (9c)  $E$  denotes a contribution proportional to the Coulomb interaction energy of a nucleus. A selection of the values of the specific elementary charges (9) is presented in table 1 (which is adapted from [20]).

From equations (7) and (8) we conclude that the theoretically expected composition dependence of  $\hat{\delta}_A$  is of the form

$$\hat{\delta}_A = \alpha_0 + \sum_{i=1}^n \alpha_i \xi_A^i \quad (10)$$

where  $\alpha_0 \equiv h(\beta_0 + \sum_j \beta_j \xi_E^j)\beta_0$ ,  $\alpha_i \equiv h(\beta_0 + \sum_j \beta_j \xi_E^j)\beta_i$ .

The  $A$ -independent contribution  $\alpha_0$  in equation (10) is not accessible from the measurements of equation (6). The best we can hope for is to measure the  $n$  effective

† 'Specific' is used here in the sense of 'per unit (atomic) mass'.

**Table 1.** A selection of possible proof-mass materials and their corresponding specific elementary charges. Neutron numbers and masses are averages weighted with natural isotope abundances.

Element	Z	N	$\left(\frac{N+Z}{\mu} - 1\right) 10^3$	$\frac{N-Z}{\mu}$	$\frac{Z(Z-1)}{(N+Z)^{1/3}\mu}$
Be	4	5	-1.351 75	0.110 961	0.640 133
C	6	6.011	-0.003 072	0.000 916	1.090 64
Mg	12	12.320 2	0.623 22	0.013 174	1.874 51
Al	13	14	0.684 212	0.037 062	1.927 24
Si	14	14.108 7	0.825 719	0.003 870	2.131 29
Ti	22	25.93	1.077 2	0.082 083	2.656 44
V	23	27.997 5	1.099 87	0.098 103	2.678 53
Cu	29	34.616 6	1.116 63	0.088 387	3.200 96
Ge	32	40.71	1.070 46	0.119 919	3.272 28
Zr	40	51.318 4	1.038 7	0.124 073	3.797 5
Ag	47	60.963 2	0.881 352	0.129 447	4.209 24
Sn	50	68.807 9	0.822 075	0.158 435	4.198 19
Ba	56	81.421 6	0.689 875	0.185 118	4.346 2
Ta	73	108	0.287 415	0.193 425	5.135 02
W	74	109.898	0.266 057	0.195 257	5.166 96
Pt	78	117.116	0.182 95	0.200 511	5.308 13
Au	79	118	0.169 856	0.198 003	5.376 59
Bi	83	126	0.093 913	0.205 761	5.487 88
U	92	146	-0.213 316	0.226 842	5.675 02

coupling parameters  $\alpha_i$ . Once the  $\alpha_i$  are known, it will be possible to measure the fundamental coupling parameters  $h$ ,  $\beta_0$ ,  $\beta_i$  (modulo the rescaling freedom  $\beta_0 \rightarrow \lambda\beta_0$ ,  $\beta_i \rightarrow \lambda\beta_i$ ,  $h \rightarrow \lambda^{-2}h$ ) if and only if one knows (or assumes) something about the relative value of  $\beta_0$  with respect to the  $\beta_i$ . Such a knowledge is available both in vector models ( $\beta_0 = 0$ ) and in string-scalar ones (see equation (6.13) of the first reference in [17]). Note that the composition of the Earth only enters by introducing an  $A$ -independent proportionality factor between the  $\alpha_i$  and the  $\beta_i$ , and does not influence the strategy of choice of the configuration  $\mathcal{C}$ . The nonlinearity of the relation  $\alpha_i = \alpha_i(\beta_j)$  can be tackled after having extracted (by least-squares fit) the  $\alpha$ 's from the raw measurements.

Given the form (10), what is the optimal choice of materials? This question has been addressed in [20]. Assuming that the measurement of the various differential accelerations  $(\Delta a/a)_{AB}^{\text{measured}} \equiv m_{AB}$  can be modelled as containing independent Gaussian errors,  $m_{AB} = \hat{\delta}_A - \hat{\delta}_B + n_{AB}$  with  $\langle n_{AB} n_{A'B'} \rangle = \sigma_{AB}^2 \delta_{A'B'}$ , one defines the likelihood function of the theory parameters  $\alpha_i$  (given a data set on some configuration  $\mathcal{C}$  of pairs of materials)

$$\chi^2(\alpha_i) = \sum_{AB \in \mathcal{C}} \sigma_{AB}^{-2} (\hat{\delta}_A - \hat{\delta}_B - m_{AB})^2 = \sum_{AB} \sigma_{AB}^{-2} \left( \sum_i \alpha_i \xi_{AB}^i - m_{AB} \right)^2, \quad (11)$$

where  $\xi_{AB}^i \equiv \xi_A^i - \xi_B^i$ . The minimum of the function  $\chi^2(\alpha_i)$  then defines the best-fit values of the  $\alpha_i$ . These  $\alpha_i^{\text{best fit}}$  are random variables (when the realization of the noise changes) with average values the true values of the  $\alpha_i$  and deviations  $\bar{\alpha}_i = \alpha_i^{\text{best fit}} - \alpha_i^{\text{true}}$  some zero-mean Gaussian variables with distribution function  $\propto \exp[-\Delta\chi^2/2]$  where  $\Delta\chi^2 = \chi^2(\alpha_i) - \chi_{\min}^2$

can be written as

$$\Delta\chi^2 = \sum_{AB \in \mathcal{C}} \sigma_{AB}^{-2} \left[ \sum_i \bar{\alpha}_i \xi_{AB}^i \right]^2 = \sum_i \sum_j g_C^{ij} \bar{\alpha}_i \bar{\alpha}_j, \tag{12}$$

where one has defined

$$g_C^{ij} \equiv \sum_{AB \in \mathcal{C}} \sigma_{AB}^{-2} \xi_{AB}^i \xi_{AB}^j. \tag{13}$$

In geometrical terms, the choice of a configuration of pairs of materials  $\mathcal{C} = \{(A_n B_n)\}$  defines a quadratic form, i.e. a *metric*  $g_C^{ij}$ , equation (13), in the  $n$ -dimensional space of coupling parameters  $\alpha_i$  ( $\alpha$ -space). The (natural, Gaussian) ellipsoids of errors of the  $\alpha_i$  are centred around  $\alpha_i^{\text{true}}$  and are defined by the above quadratic form:  $\sum_{ij} g_C^{ij} \bar{\alpha}_i \bar{\alpha}_j \leq \Delta\chi^2 = \text{constant}$ , where the value of  $\Delta\chi^2$  depends both on  $n$  and on the chosen level of confidence (e.g. if  $n = 2$  the ellipsoids  $\Delta\chi^2 = 2.3$  and  $\Delta\chi^2 = 6.2$  correspond to 68% and 95% confidence levels, respectively). Optimizing the choice of configuration  $\mathcal{C}$  means choosing a set of ‘connecting vectors’  $\xi_{AB}^i \equiv \xi_A^i - \xi_B^i$  in the  $n$ -dimensional  $\xi$ -space of specific elementary charges such that the corresponding quadratic form (13) defines the smallest and ‘roundest’ ellipsoids in the dual  $\alpha$ -space. A general geometrical rule for achieving this is to choose the connecting vectors  $\xi_{AB}^i$  so as to span the largest and least degenerate (i.e. as far as possible from  $(n - 1)$ -dimensional configurations) volume in  $\xi$ -space. Minimal configurations are made of  $n$  vectors. The volume of the ellipsoid of errors is inversely proportional to the volume  $\varepsilon_{ij\dots\ell} \xi_1^i \xi_2^j \dots \xi_n^\ell$  spanned by the  $n$  vectors, and its shape is determined by the shape of the vectorial configuration  $\vec{\xi}_1, \vec{\xi}_2, \dots, \vec{\xi}_n$ . (Note that the geometry of  $\xi$ -space is purely *affine*, i.e. does not make use of the concepts of (Euclidean) length or angle.) To illustrate this geometrical approach, we represent in figure 1, using table 1, the three-dimensional  $\xi$ -space defined by equations (9). See [20] for an application to the choice of a configuration of pairs of materials.

The strategy just explained is appropriate to an ambitious experiment (such as M3 STEP) which aims, at once, to establish convincingly the existence of an EP violation, and to

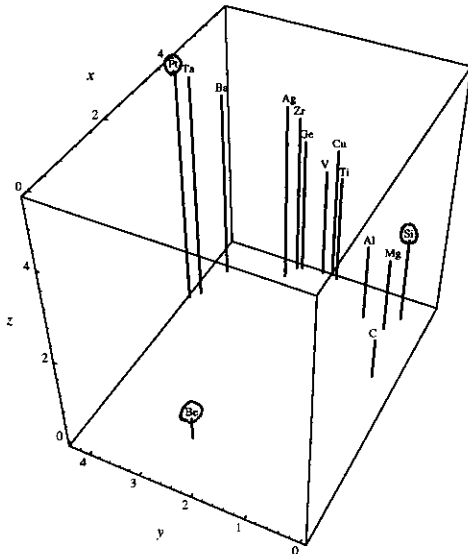


Figure 1. Position of the elements in the three-dimensional  $\xi$ -space of specific elementary charges. The  $x$ ,  $y$  and  $z$  axes are proportional to  $\xi^1 - 1$ ,  $\xi^2$  and  $\xi^3$  of equations (9).

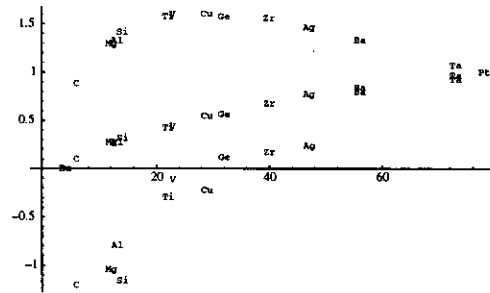
maximize the precision of the simultaneous measurement of the underlying theoretical coupling parameters. Such an experiment requires a minimum of four differential accelerometers: three of them spanning as large a volume as possible in  $\xi$ -space, and the fourth one providing a redundancy check (closing a polygon or repeating an edge). One can, however, settle for less ambitious strategies if one considers descoped versions of STEP. I now wish to introduce such a strategy. The basic idea is to argue that theoretical expectations suggest plausible relative orders of magnitude for the coupling parameters  $\beta_i$  (and thereby  $\alpha_i$ ), and thereby put constraints on the composition dependence of the  $\hat{\delta}_A$ .

First, in vector models it is plausible that the couplings to baryon number and lepton number be of the same order of magnitude, i.e.  $\beta_1 \sim \beta_2$  and therefore  $\alpha_1 \sim \alpha_2$ . An example of this is provided by grand unified theories which predict [14] a coupling to  $B - L = N = \frac{1}{2}(N + Z) + \frac{1}{2}(N - Z)$ . As, numerically,  $\xi^1 = B/\mu$  varies much less over the periodic table than  $\xi^2 = (N - Z)/\mu$  (see table 1), we conclude that, in most cases, the vector-specific charge can be well approximated by  $\xi^2$  or, nearly equivalently, by  $N/\mu$ . We should, however, also consider the possibility that the vector charge is *exactly* proportional to the baryon number. In other words, we argue that the one-dimensional continuum of possible values of  $\alpha_1/\alpha_2$  can be, in a first approximation, reduced to a dichotomy between a charge  $\propto (B - L)$  and a charge  $\propto B$ .

Second, in string-inspired scalar models one has put plausible constraints on the magnitudes of  $\beta_1$  and  $\beta_2$  (see [17], p 552) which are such that the composition dependence in equation (3) is numerically dominated by the last term, i.e. by the coupling to the nuclear electrostatic energy. In other words, we argue that the two-dimensional continuum of possible values of  $\alpha_1/\alpha_3$ ,  $\alpha_2/\alpha_3$  can be, in a first approximation, reduced to a unique coupling  $\propto E$ . We have checked numerically the above assertions by plotting the variation over the periodic table ( $Z$  dependence) of  $\hat{\delta}_A$  (normalized by imposing  $\hat{\delta}_{\text{Be}} = 0$  and  $\hat{\delta}_{\text{Au}} = 1$ ) in various theoretical models: couplings to  $L$ ,  $B - L$ ,  $B \pm 3L$ ,  $E$  and  $E \pm 240B \pm 5.2L$  (the latter coefficients being the maximal ones suggested in [17]). Instead of spreading all over the plane, the above curves gather themselves into two well separated bundles of curves centred around the curves defined by couplings to  $L$  and  $E$ .

Our conclusion is that, in a first approximation (i.e. barring values of the coupling parameters that do not appear *a priori* justified within the framework of present theoretical ideas), the search among the continuum of theoretical models can be simplified into a search among only *three* basic possibilities: a coupling to  $L$  (or  $B - L$ ), a coupling to  $B$  or a coupling to  $E$ . This trichotomy is illustrated in figure 2 which presents the corresponding  $Z$  dependences of the EP violation signals  $\hat{\delta}_A$  (normalized by an affine transformation  $\hat{\delta}_A^{\text{new}} = a\hat{\delta}_A^{\text{old}} + b$  so that  $\hat{\delta}_{\text{Be}}^{\text{new}} = 0$  and  $\hat{\delta}_{\text{Au}}^{\text{new}} = 1$ ). We conclude from figure 2 that, in order to distinguish between theoretical models, it is very important to include, among the tested materials; beryllium, platinum (or gold) and an element among {C, Mg, Si, Al}, say silicon (maybe in the form of silica  $\text{SiO}_2$ ). (Magnesium is probably too reactive to be considered seriously.) This leads to a minimum of *two* pairs connecting the three elements Be, Pt, Si, to which must be added a third, redundant pair (e.g. a repeated (Be Pt)) to include a theory-independent check on the reality of EP violation.

In all the above strategies, one tries to get some definite information about the theoretical nature of an EP violation. If one has to descop a space mission to the minimum meaningful concept, one can go down to *two* pairs of materials containing the same materials, say the pairs  $(AB)$ ,  $(AB)'$ , where the prime indicates that some difference (in shape, mass, etc) is introduced. The choice  $A = \text{Be}$ ,  $B = \text{Pt}$  seems appropriate for maximizing the expected signal in most models (see figure 2). Such a trimmed concept is appropriate to a discovery



**Figure 2.** The observable (normalized) violation of the equivalence principle  $\delta_A^{new}$  as a function of atomic number  $Z$  plotted for couplings to  $B$  (upper curve),  $E$  (intermediate curve) and  $B-L$  (lower curve). The latter two curves are typical representatives of large continua of (respectively) vector and scalar models having coupling parameters of a naturally expected order of magnitude.

experiment where one puts the emphasis on establishing (by a redundancy) the reality of an EP violation, rather than on extracting theoretical information from the data.

### 3. Conclusions

The main points emphasized above can be summarized as follows.

- In spite of its impressive name, and of its having been put on a pedestal by part of the ‘relativity literature’, the ‘equivalence principle’ is not at all a basic taboo principle of physics. On the contrary, nearly all the attempts to extend the present framework of physics (Kaluza–Klein, strings, supersymmetry, etc) predict the existence of new interactions (mediated by scalar or vector fields) violating the universality of free fall.
- Though present tests of the universality of free fall are at the  $10^{-12}$  level, there exist string-inspired models (containing no small parameters) in which the theoretically expected level of EP violation is naturally  $\ll 10^{-12}$ . Equivalence principle tests are, by far, the most sensitive low-energy probes of such new physics beyond the present framework.
- Present theoretical models suggest a rationale for optimizing the choice of pairs of materials to be tested in EP experiments. In descoped experiments, one can trim this general rationale down to a simple trichotomy among competing classes of theoretical models. This leads to comparing Be, Pt and Si (or  $\text{SiO}_2$ , or C, or Al). In a discovery experiment, one can use only two pairs  $(AB)$ ,  $(AB)'$  (with some difference).

### References

- [1] Einstein A 1907 *Jahrbuch Radioaktiv.* **4** 411
- [2] Einstein A 1911 *Ann. Phys., Lpz.* **35** 898
- [3] Su Y *et al* 1994 *Phys. Rev. D* **50** 3614
- [4] Dickey J O *et al* 1994 *Science* **265** 482  
Williams J G, Newhall X X and Dickey J O 1996 *Phys. Rev. D* **53** 6730
- [5] Reasenberg R D *et al* 1979 *Astrophys. J.* **234** L219  
Hellings R W 1984 *General Relativity and Gravitation* ed B Bertotti *et al* (Dordrecht: Reidel) pp 365–85; see also [4] and [8]
- [6] Taylor J H 1994 *Rev. Mod. Phys.* **66** 711
- [7] Taylor J H *et al* 1992 *Nature* **355** 132  
Damour T and Taylor J H 1992 *Phys. Rev. D* **45** 1840



- [8] Will C M 1993 *Theory and Experiment in Gravitational Physics* (Cambridge: Cambridge University Press)
- [9] Kaluza K 1921 *Sitz. Preuss. Akad. Wiss.* 966
- [10] Jordan P 1949 *Nature* **164** 637; 1955 *Schwerkraft und Weltall* (Braunschweig: Vieweg); 1959 *Z. Phys.* **157** 112
- [11] Scherk J and Schwarz J H 1974 *Phys. Lett.* **52B** 347
- [12] Scherk J 1979 *Phys. Lett.* **88B** 265
- [13] Fayet P 1977 *Phys. Lett.* **69B** 489; 1986 *Phys. Lett.* **171B** 261; **172B** 363
- [14] Fayet P 1996 *Class. Quantum Grav.* **13** A19–31
- [15] Fierz M 1956 *Helv. Phys. Acta* **29** 128
- [16] Brans C and Dicke R H 1961 *Phys. Rev.* **124** 925
- [17] Damour T and Polyakov A M 1994 *Nucl. Phys. B* **423** 532; *Gen. Rel. Grav.* **26** 1171
- [18] Damour T and Vokrouhlický D 1996 *Phys. Rev. D* **53** 4177; available on gr-qc
- [19] Damour T and Esposito-Farèse G 1992 *Class. Quantum Grav.* **9** 2093
- [20] Damour T and Blaser J P 1994 *Particle Astrophysics, Atomic Physics and Gravitation* ed J Tran Than Van *et al* (Gif-sur-Yvette: Editions Frontières) pp 433–40
- [21] Blaser J P 1996 *Class. Quantum Grav.* **13** A87–90
- [22] Lockerbie N 1996 *Class. Quantum Grav.* **13** A53–7
- [23] Touboul P *et al* 1996 *Class. Quantum Grav.* **13** A67–78