APO Telescope Coordinates

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1 Introduction

In order to make a reliable prediction of the range to the moon from APO, we need to know where on the earth APO sits. Latitudes are generally reported as geodetic latitudes, which refer to the angle the surface normal (of the reference ellipsoid) makes with the equatorial plane. This is slightly different than the angle to the site from the earth's center, and for APO differs by about $0.17°$. Geodetic latitude is always larger than geocentric, due to the oblate geometry of the earth.

Geodetic coordinates are referred to a reference ellipsoid: a matheatically simple surface that approximates the geoid. The geoid is an equipotential surface corresponding to mean sea level. It is the mean surface of the ocean, extended under land to the height that sea water would establish in a frictioness network of tubes. The geoid departs from the reference ellipsoid by as much as -100 m near Sri Lanka to $+60$ m in the North Atlantic. The geodetic height of a point on the earth is measured as the perpendicular distance from the reference ellipsoid.

2 Geodetic Coordinates of APO

The White Sands Missle Range once performed an experiment using the 3.5 meter telescope, involving tracking rocket launches. They established the position of the telescope mount using differential GPS techniques, placing the antenna on top of the dome, plumbed to be directly above the telescope axis intersection. A precision (millimeter) taping operation measured the vertical distance between the antenna phase center and the telescope axis. A conversation with a member of the survey team convinces me that they obtained an accuracy of about 0.1 m. The original coordinates expressed in the Jan 1996 report have since been modified to reflect adjustments to the coordinates of the base stations in the valley.

The results are expressed in the WGS84 (World Geodetic System, 1984) coordinate system, which defines the reference ellipsoid to have a semi-major axis of $a = 6378.137$ km, and a flattening factor, $f \equiv \frac{a-b}{a} = \frac{1}{298.257223563}$, where a and b are the semi-major and semi-minor axes of the ellipsoid, respectively. The reported coordinates for the 3.5 meter telescope are then:

$$\begin{aligned} \phi &= 32.78035394° \\ \lambda &= 254.1795786° \\ h &= 2786.62\,\mathrm{m} \end{aligned}$$

3 Conversion to Geocentric Coordinates

Conversion to geocentric coordinates can be accomplished via the following relations:

$$Z = \rho \sin\phi' = \left[N_\phi(1-e^2) + h\right]\sin\phi, \text{ and}$$

$$\sqrt{X^2+Y^2} = \rho\cos\phi' = (N_\phi + h)\cos\phi,$$

where $e^2 = 2f - f^2$, and

$$N_\phi = \frac{a}{\sqrt{1-e^2\sin^2\phi}}.$$

Here, ϕ' is the geocentric latitude, and ρ is the distance to the center of the earth. The X, Y, and Z coordinates are the geocentric cartesian coordinates of the point. For the purpose of calculating the gecentric latitude, it is unnecessary to resolve X and Y, which is easily done via the site longitude.

4 Geocentric Coordinates of APO

Running the geodetic latitude and height through the above calculations, we arrive at the APO 3.5 meter geocentric coordinates:

$$\begin{aligned}
\rho\cos\phi' &= 5370.04530 \\
\rho\sin\phi' &= 3435.01285 \\
\rho &= 6374.69213 \\
\phi' &= 32.6054942^\circ \\
\lambda &= 254.1795786^\circ
\end{aligned}$$

5 Lunar-derived Coordinates

Ultimately, the lunar range data will provide a far more precise set of geocentric coordinates for the telescope. When this happens, this section of this document will be modified to contain these results.